Dynamic Longitudinal Models for Criminological Panel Data

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Introduction

- Application of continuous-time models for exploring the association between victimization and offending of adolescents (victim-offender overlap)
- Theoretical framework: Lifestyle-routine activity approach (Cohen & Felson, 1979; Hindelang et al., 1978)
- Lifestyle and routine activities (particularly with friends) are strong correlates of both victimization and offending
- Structured versus unstructured socializing
  - Structured activities reduce the risk for involvement in offending and/or victimization (e. g., activities with parents, participating in sports event)
  - Unstructured activities increase the risk for offending/victimization (e. g., going out at night, drinking alcohol)

Study and Data

- Longitudinal panel data from the study *Crime in the Modern City* (Crimoc, `www.crimoc.org`, Boers et al., 2010)
- The study has 13 panel waves covering the age range from 13 to 30. Data was conducted in Duisburg.
- 7 waves cover the period from 14 to 20 years of age.
- All participants with maximum of two missing panel waves are considered (participation in at least 5 out of 7 waves; n=2679)
- Previous analyses of the victim-offender overlap:
  - Erdmann & Reinecke (2018): Growth Curve Models, Cross-Lagged Panel Model
  - Erdmann & Reinecke (2019): Mixture Models
  - Erdmann (2020): Structural Equation Mediation Models

Measurements - Dependent Variables

- **Offending**: Index of 15 items measuring delinquency including violence (robbery, violent bag snatching, assault with a weapon, assault without a weapon), property offenses (shoplifting, burglary, theft of bicycles, theft of cars, theft out of cars, theft out of a vending machine, fencing, other theft), and criminal damage offenses (graffiti, scratching, property damage)
- **Victimization**: Index of 3 items measuring violent victimization (robbery with threat of violence, assault with a weapon, and assault without a weapon)
- For both measurements summed annual incidences (0-12+) are calculated.

Measurements - Independent Variables

- **Gender** (male/female)
- Routine activities with friends
  - **Frequency of meeting** with friends
  - **Partying** (Visiting bars/discotheques, drinking a lot of alcohol)
  - **Cultural activities** (playing theatre, making music, working on a newspaper)
  - **Studying**

In line with previous literature, values for activities were averaged over the the 7-year-period (Labouvie, Pandina, & Johnson, 2016; Mulford et al., 2018)

Descriptive Results

	Victimization			Offending		
Age	n	Mean	Var.	n	Mean	Var.
14	2201	0.61	4.05	2208	2.05	14.96
15	2406	0.47	2.89	2422	1.98	15.22
16	2563	0.40	2.45	2568	1.54	12.31
17	2480	0.36	2.25	2484	1.19	9.94
18	2435	0.23	1.53	2436	0.71	5.84
19	2455	0.15	0.84	2457	0.47	3.70
20	2436	0.11	0.54	2439	0.35	2.80

Note. Mean/Variance incidence for violent victimization and general offending, based on seven-wave panel, $n = 2679$, maximum of two missing wave information, full information maximum likelihood for estimating means and variances, and values rounded to two decimal digit.

Descriptive Results

Gender	n	Proportion	
female	1469	0.55	
male	1207	0.45	
Activities	**n**	**Mean**	**Var.**
Frequency meeting	2597	2.97	0.43
Partying	2600	2.66	0.89
Cultural activities	2597	1.57	0.55
Studying	2598	2.71	0.88

Note. Proportions of gender and means/variances for group activities, based on seven-wave panel, $n = 2679$, maximum of two missing wave information, full information maximum likelihood for estimating means and variances, and values rounded to two decimal digit.

Continuous Time Model (Victim-Offender-Relation)

Initial latent state:

$$\underbrace{\begin{bmatrix}\text{off}\\ \text{vikt}\end{bmatrix}(t_0)}_{\eta(t_0)} \sim \mathrm{N}\left(\underbrace{\begin{bmatrix}\text{T0m\_off}\\ \text{T0m\_vikt}\end{bmatrix}}_{\text{T0MEANS}}, \underbrace{\mathit{covsdcor}\left\{\begin{bmatrix}\text{T0var\_off} & 0\\ \text{T0var\_vikt\_off} & \text{T0var\_vikt}\end{bmatrix}\right\}}_{\mathbf{Q}^{*}{}_{t0}(\text{T0VAR})}\right)$$

- *covsdcor* = transposed cross product of *cholsdcor*, to give covariance.
- *cholsdcor* converts lower triangular matrix of standard deviation and unconstrained correlation to Cholesky factor covariance, see Driver & Voelkle (2018: 11).

Continuous Time Model (Victim-Offender-Relation)

Deterministic change:

$$\mathrm{d}\begin{bmatrix}\text{off}\\ \text{vikt}\end{bmatrix}(t) = \left(\begin{bmatrix}\text{drift\_off} & \text{drift\_off\_vikt}\\ \text{drift\_vikt\_off} & \text{drift\_vikt}\end{bmatrix}\begin{bmatrix}\text{off}\\ \text{vikt}\end{bmatrix}(t) + \begin{bmatrix}\text{cint1}\\ \text{cint2}\end{bmatrix}\right)\mathrm{d}t \quad +$$

Random change:

$$\mathit{cholsdcor}\left\{\begin{bmatrix}\text{diff\_off} & 0\\ \text{diff\_vikt\_off} & \text{diff\_vikt}\end{bmatrix}\right\}\mathrm{d}\begin{bmatrix}W_1\\ W_2\end{bmatrix}(t)$$

$$\mathrm{d}\eta(t) = \underbrace{A}_{\text{DRIFT}}\eta_t + \underbrace{b}_{\mathit{CINT}} + \underbrace{\mathbf{G}}_{\text{DIFFUSION}}\mathrm{d}\mathbf{W}(t)$$

- *cholsdcor* converts lower triangular matrix of standard deviation and unconstrained correlation to Cholesky factor covariance, see Driver & Voelkle (2018: 11).

Continuous Time Model (Victim-Offender-Relation)

Observations:

$$\underbrace{\begin{bmatrix}\text{off}\\ \text{vikt}\end{bmatrix}(t)}_{\mathbf{Y}(t)} = \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\tilde{}\text{(LAMBDA)}} \underbrace{\begin{bmatrix}\text{off}\\ \text{vikt}\end{bmatrix}(t)}_{\eta(t)} + \underbrace{\begin{bmatrix}0\\ 0\end{bmatrix}}_{\tau\text{(MANIFESTMEANS)}} + \underbrace{\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}}_{\theta\text{(MANIFESTVAR)}} \underbrace{\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\end{bmatrix}(t)}_{\varepsilon(t)}$$

Latent noise per time step:

$$\Delta\left[W_{j\in[1,2]}\right](t-u) \sim \mathrm{N}(0, t-u)$$

Observation noise:

$$\left[\varepsilon_{j\in[1,2]}\right](t) \sim \mathrm{N}(0, 1)$$

CTM Results based on wide-format data

Log-Likelihood and Information Criteria for Model Comparison

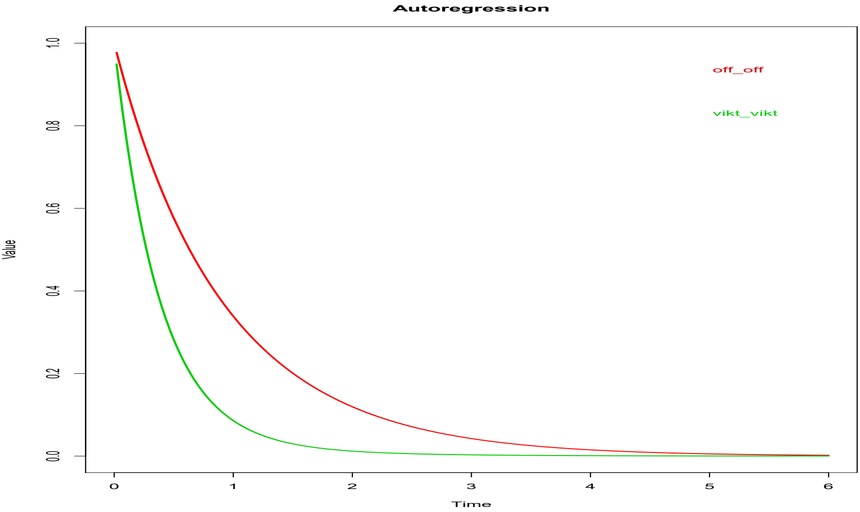
Unconditional Models	Par.	- 2 log(L)	AIC	BIC
Model 1 (ctsem: omx) (no trait variances)	14	140408.1	72456.1	-127771.2
Model 2 (ctsem: omx) (trait variances)	17	140191.4	72245.35	-127964.3
Conditional Models				
Model 3 (ctsem: omx) (no trait variances)	44	167831.67	73803.67	-203259.2
Model 4 (ctsem: omx) (trait variances)	47	167678.03	73656.03	-203389.1

CTM Results based on wide-format data

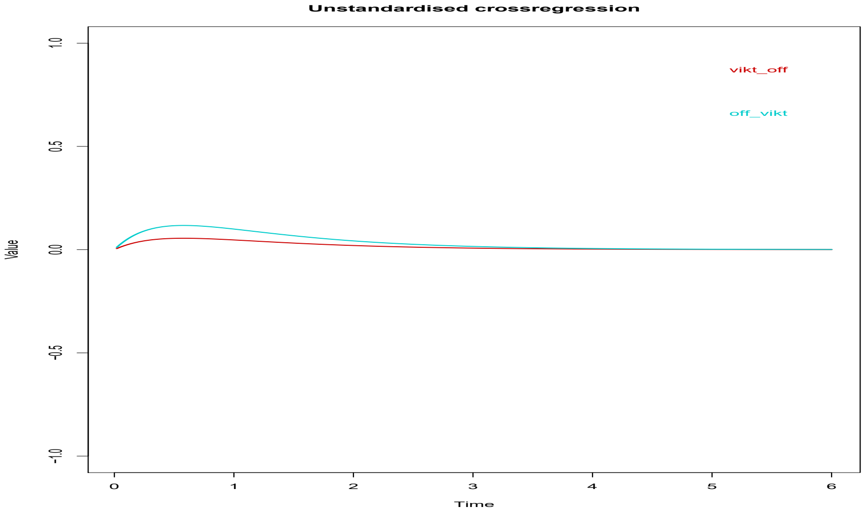
Parameters of the unconditional models

		Model 1		Model 2	
Matrix/Vector	Parameter	Estimate	SE	Estimate	SE
Drift Matrix (A)	$a_{off,off}$	-0.902	0.019	-1.132	0.038
	$a_{vikt,vikt}$	-1.835	0.050	-2.580	0.136
	$a_{vikt,off}$	0.202	0.015	0.267	0.034
	$a_{off,vikt}$	0.451	0.060	0.571	0.011
CT Intercepts (b)	$cint1$	0.623	0.036	0.856	0.056
	$cint2$	0.272	0.027	0.433	0.048
Trait Variance (ϕ_ξ)	$\phi_{off,off}$	−−	−−	1.262	0.129
	$\phi_{vikt,vikt}$	−−	−−	0.214	0.018
	$\phi_{off,vikt}$	−−	−−	0.265	0.037
Diffusion Matrix (Q)	q_{off}	13.845	0.238	14.512	0.336
	q_{vikt}	5.957	0.159	7.521	0.358
	$q_{vikt,off} = q_{off,vikt}$	0.423	0.136	0.296	0.235
Initial Occasion (t_0)	$\mathrm{M}(off_{t0})$	2.219	0.083	2.209	0.081
	$\mathrm{M}(vikt_{t0})$	0.656	0.043	0.655	0.043
	$\mathrm{var}(off_{t0})$	16.108	0.506	15.235	0.490
	$\mathrm{var}(vikt_{t0})$	4.162	0.129	3.884	0.124
	$\mathrm{cov}(vikt_{t0}, off_{t0})$	2.676	0.192	2.158	0.182

Model 2: Autoregression Plot



Model 2: Crossregression Plot



CTM Results based on wide-format data

Parameters of the unconditional models

	Model 3		Model 4	
Parameter (TIPREDEFFECT)	Estimate	SE	Estimate	SE
$b_{off,gender}$	0.456	0.063	**0.513**	0.076
$b_{off,meetingfriends}$	0.361	0.047	**0.409**	0.058
$b_{off,partying}$	0.413	0.035	**0.463**	0.043
$b_{off,cultural}$	0.044	0.041	0.051	0.048
$b_{off,studying}$	-0.278	0.035	**-0.313**	0.043
$b_{vikt,gender}$	0.124	0.047	0.165	0.073
$b_{vikt,meetingfriends}$	0.063	0.035	0.077	0.055
$b_{vikt,partying}$	0.137	0.026	0.187	0.041
$b_{vikt,cultural}$	-0.004	0.030	-0.007	0.047
$b_{vikt,studying}$	-0.054	0.026	-0.072	0.041

Discussion of the wide-format results

- The results have shown that it is necessary to consider stable between-subject differences (trait differences) in discrete as well as in continuous time models. The opportunity to estimate latent trait variances and covariances in continuous time models is comparable with the extension of random intercepts in discrete time models.
- The diagonal elements of the drift matrix (Matrix A) and the autoregression plots show that victimization is less stable compared to offending.

Discussion of the wide-format results

- The off-diagonal elements of the drift matrix and the crossregression plots indicate that the impact of victimization on offending is stronger than the impact of offending on victimization. This impact holds for the phase of early adolescence (14 to 16 years of age) but tends to diminish later.
- The impact of gender is stronger on offending compared to the impact on victimization. Risky activities like hanging around with friends have a stronger impact on offending compared to victimization. On the other hand unrisky activities like studying together have a reasonable negative impact on offending and on victimization.

CTM based on Stan-functions (long-format data)

Step	`ctsem`-command	Type
Specification	`ctModel`	'stanct'
Fitting	`ctStanFit`	nopriors=T (ML) priors=T (ML)
Summary	`summary` `ctModelLatex`	

Plotting	`ctStanDiscretePars`
	`ctKalman`
	`ctStanTIpredeffect`
		plot=T
		plot=T
		whichpars=c('DRIFT')
		whichpars=c('CINT')

CTM Results based on long-format data

Log-Likelihood and Information Criteria for Model Comparison

Uncond./Conditional Model	Par.	- 2 log(L)	AIC	BIC
Model A (ctsem: omx)	14	140408.1	72456.1	-127771.2
Model B (ctsem: omx) (Gender)	18	144079.8	70783.77	-145190.2

Uncond./Conditional Models	Par.	- log(L)	AIC	BIC
Model C1 (ctsem: stanct) (nopriors)	21	-70032.27	140106.5	– –
Model C2 (ctsem: stanct) (priors)	21	-70032.41	140106.8	– –
Model D1 (ctsem: stanct) (nopriors, Gender)	32	-68805.88	137675.8	– –
Model D2 (ctsem: stanct) (priors, Gender)	32	-68806.36	137676.7	– –

CTM Results based on long-format data

Log-Likelihood and Information Criteria for Model Comparison

Rstricted Models	Par.	- log(L)	AIC	BIC
Model D3 (ctsem: stanct) (nopriors, Gender)	31	-68807.14	137676.3	——
Model D4 (ctsem: stanct) (priors, Gender)	31	-68807.85	137677.7	——

```
TOMEANS vector spec input rowwise into 2 * 1 matrix:
[,1]
[1,]  "T0m_off |||| Gender"
[2,] "T0m_vikt |||| Gender"
LAMBDA vector spec input rowwise into 2 * 2 matrix:
[,1] [,2]
[1,]  "1"  "0"
[2,]  "0"  "1"
```

CTM Results based on long-format data

Log-Likelihood and Information Criteria for Model Comparison

```
DRIFT vector spec input rowwise into 2 * 2 matrix:
[,1]                         [,2]
[1,]      "drift_off |||| Gender"         "drift_off_vikt"
[2,] "drift_vikt_off |||| Gender" "drift_vikt |||| Gender"
DIFFUSION vector spec input rowwise into 2 * 2 matrix:
[,1]                        [,2]
[1,]      "diff_off |||| Gender"         "diff_off_vikt"
[2,] "diff_vikt_off |||| Gender" "diff_vikt |||| Gender"
CINT vector spec input rowwise into 2 * 1 matrix:
[,1]
[1,] "cint1 |||| Gender"
[2,]             "cint2"
```

CTM Results based on long-format data

`ctsem`, nopriors, Model C1

Initial latent state:

$$\underbrace{\begin{bmatrix}\text{off}\\ \text{vikt}\end{bmatrix}(t_0)}_{\eta(t_0)} \sim \mathrm{N}\left(\begin{bmatrix}2.224\\ 0.657\end{bmatrix}, \underbrace{\begin{bmatrix}0.402 & 0\\ 0.285 & 0.204\end{bmatrix}}_{\mathbf{Q}^*{}_{t0}}\right)$$

Deterministic change:

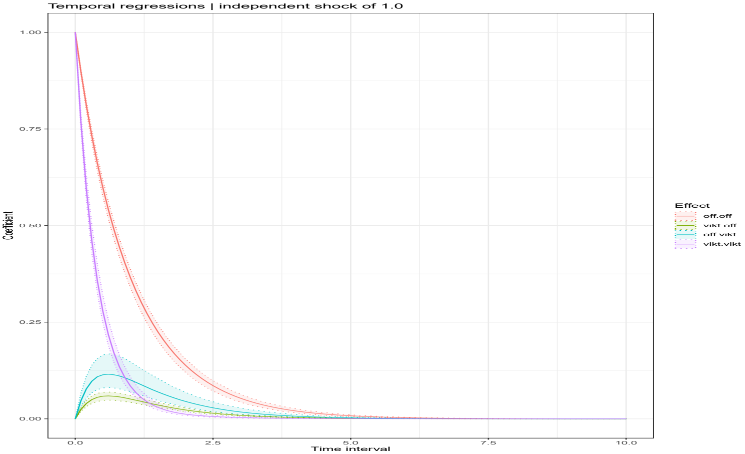
$$\underbrace{\mathrm{d}\begin{bmatrix}\text{off}\\ \text{vikt}\end{bmatrix}(t)}_{\mathrm{d}\eta(t)} = \left(\underbrace{\begin{bmatrix}-1.06 & 0.564\\ 0.284 & -2.586\end{bmatrix}}_{\mathbf{A}}\underbrace{\begin{bmatrix}\text{off}\\ \text{vikt}\end{bmatrix}(t)}_{\eta(t)} + \underbrace{\begin{bmatrix}0.768\\ 0.417\end{bmatrix}}_{\mathbf{b}}\right)\mathrm{d}t \quad +$$

Random change:

$$\underbrace{\mathit{cholsdcor}\left\{\begin{bmatrix}3.878 & 0\\ 0.027 & 2.743\end{bmatrix}\right\}}_{\mathbf{G}}\underbrace{\mathrm{d}\begin{bmatrix}W_1\\ W_2\end{bmatrix}(t)}_{\mathrm{d}\mathbf{W}(t)}$$

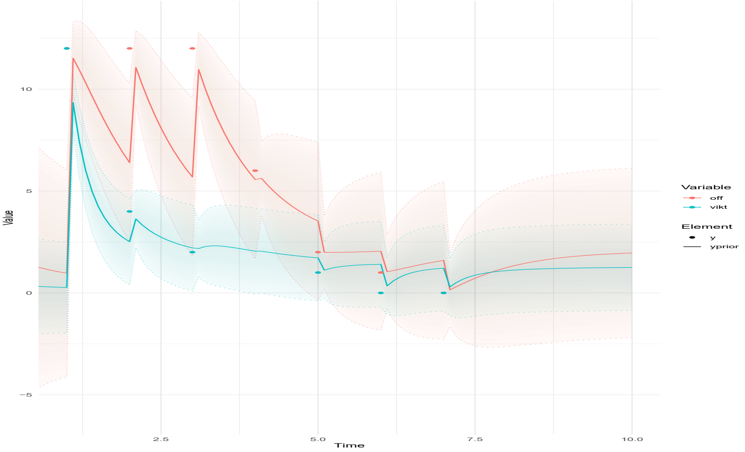
CTM Results based on long-format data

Discrete-time auto- and cross-effect dynamics



CTM Results based on long-format data

Predictions for n=5 subjects



CTM Results based on long-format data

`ctsem`, nopriors, Model D1

Initial latent state:

$$\underbrace{\begin{bmatrix}\text{off}\\\text{vikt}\end{bmatrix}(t_0)}_{\eta(t_0)} \sim \mathrm{N}\left(\begin{bmatrix}1.839\\0.448\end{bmatrix}, \underbrace{\begin{bmatrix}0.399 & 0\\0.24 & 0.202\end{bmatrix}}_{\mathbf{Q}^*{}_{t0}}\right)$$

Deterministic change:

$$\underbrace{\mathrm{d}\begin{bmatrix}\text{off}\\\text{vikt}\end{bmatrix}(t)}_{\mathrm{d}\eta(t)} = \left(\underbrace{\begin{bmatrix}-1.1154 & 0.72\\0.419 & -3.373\end{bmatrix}}_{\mathbf{A}}\underbrace{\begin{bmatrix}\text{off}\\\text{vikt}\end{bmatrix}(t)}_{\eta(t)} + \underbrace{\begin{bmatrix}0.421\\0.402\end{bmatrix}}_{\mathbf{b}}\right)\mathrm{d}t \quad +$$

Random change:

$$\underbrace{\mathit{cholsdcor}\left\{\begin{bmatrix}3.078 & 0\\-0.034 & 2.644\end{bmatrix}\right\}}_{\mathbf{G}}\underbrace{\mathrm{d}\begin{bmatrix}W_1\\W_2\end{bmatrix}(t)}_{\mathrm{d}\mathbf{W}(t)}$$

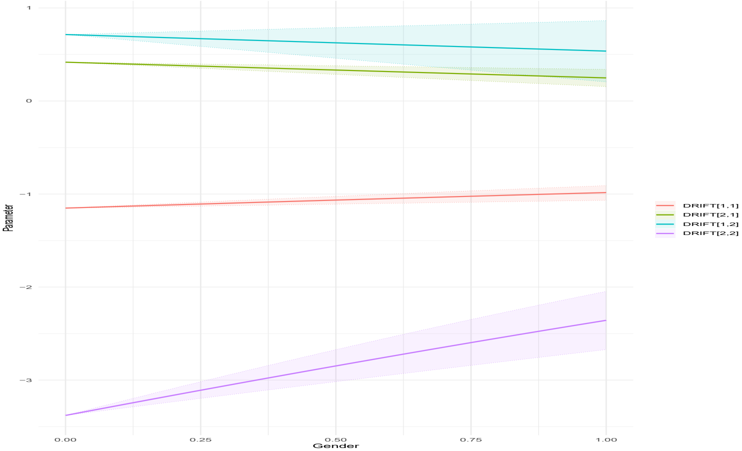
CTM Results based on long-format data

`ctsem`, nopriors, Model D1 (Matrix $tipreds)

Parameter	β	sd	2.5%	50%	97.5%	z
$a_{off,off}$	0.1793	0.0518	0.0818	0.1766	0.2847	3.4590
$a_{off,vikt}$	*-0.1872*	0.1644	-0.5133	-0.1826	0.1357	*-1.1388*
$a_{vikt,off}$	-0.1759	0.0525	-0.2888	-0.1758	-0.0762	-3.3532
$a_{vikt,vikt}$	1.1425	0.2125	0.7243	1.1417	1.5714	5.3759
$cint1$	0.7101	0.0893	0.5439	0.7106	0.8804	7.9517
$cint2$	*0.0827*	0.0891	-0.0918	0.0819	0.2527	*0.9283*
q_{off}	1.3048	0.0488	1.2124	1.3066	1.3986	26.7195
q_{vikt}	0.3629	0.0793	0.2096	0.3647	0.5126	4.5756
$q_{vikt,off}$	0.0893	0.0325	0.0267	0.0899	0.1568	2.7505
M(off_{t0})	0.8394	0.1703	0.5017	0.8367	1.1751	4.9282
M($vikt_{t0}$)	0.4635	0.0851	0.3018	0.4598	0.6285	5.4482

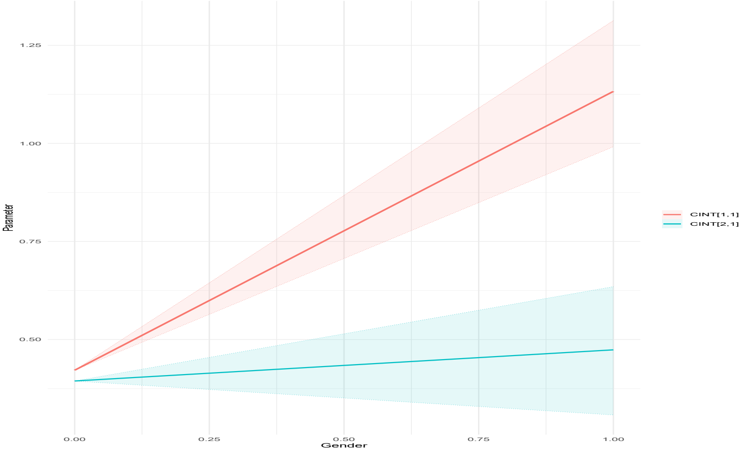
CTM Results based on long-format data

Expectations for individual parameter value change (Drift Matrix)



CTM Results based on long-format data

Expectations for individual parameter value change (Intercepts)



CTM Results based on long-format data

`ctsem`, nopriors, Model D3

Subject parameter distribution:

$$\underbrace{\begin{bmatrix} \text{T0m\_off}_i \\ \text{T0m\_vikt}_i \\ \text{cint1}_i \\ \text{cint2}_i \\ \text{drift\_off}_i \\ \text{drift\_vikt\_off}_i \\ \text{drift\_vikt}_i \\ \text{diff\_off}_i \\ \text{diff\_vikt\_off}_i \\ \text{diff\_vikt}_i \end{bmatrix}}_{\phi(i)} \approx \mathrm{N}\left(\begin{bmatrix} 1.845 \\ 0.448 \\ -1.125 \\ 0.398 \\ -3.334 \\ 3.065 \\ -0.019 \\ 2.658 \\ 0.43 \\ 0.438 \end{bmatrix}, \begin{bmatrix} 0.4 & 0 & 1.598 & 0.427 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.511 & 0.255 & 0.298 & 0.45 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.598 & 0.298 & 0.206 & -0.011 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.427 & 0.45 & -0.011 & 1.408 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \right) +$$

$$\underbrace{\begin{bmatrix} 0.831 \\ 0.461 \\ 0.697 \\ 0 \\ 0.141 \\ -0.153 \\ 1.114 \\ 1.316 \\ 0.107 \\ 0.461 \end{bmatrix}}_{\boldsymbol{\beta}} \underbrace{\begin{bmatrix} \text{Gender} \end{bmatrix}}_{\mathbf{z}}$$

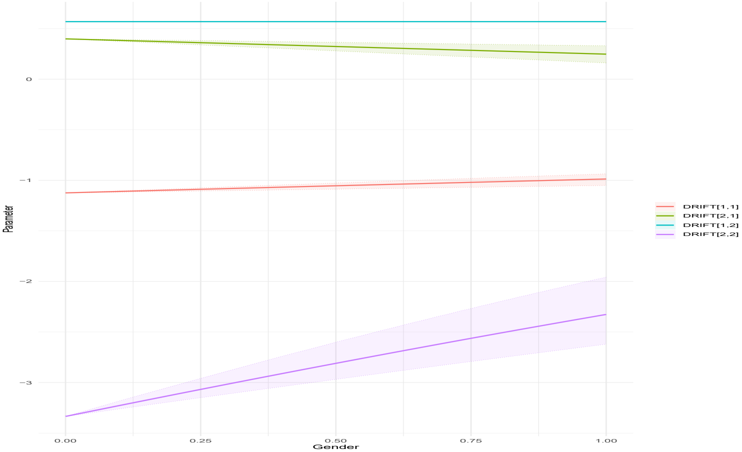
CTM Results based on long-format data

`ctsem`, nopriors, Model D3 (Matrix $tipreds)

Parameter	β	sd	2.5%	50%	97.5%	z
$a_{off,off}$	0.1413	0.0370	0.0713	0.1398	0.2186	3.8198
$a_{off,vikt}$	—	—	—	—	—	—
$a_{vikt,off}$	-0.1529	0.0523	-0.2549	-0.1521	-0.0519	-2.9236
$a_{vikt,vikt}$	1.1141	0.2120	0.7151	1.1043	1.5205	5.2559
$cint1$	0.6975	0.0834	0.5477	0.6957	0.8674	8.3676
$cint2$	—	—	—	—	—	—
q_{off}	1.3159	0.0443	1.2269	1.3180	1.3993	29.6755
q_{vikt}	0.4614	0.1229	0.1969	0.4636	0.6803	3.7541
$q_{vikt,off}$	0.1067	0.0498	0.0054	0.1086	0.2030	2.1443
$\mathrm{M}(off_{t0})$	0.8312	0.1662	0.5262	0.8250	1.1540	5.0000
$\mathrm{M}(vikt_{t0})$	0.4613	0.0855	0.2910	0.4631	0.6214	5.3950

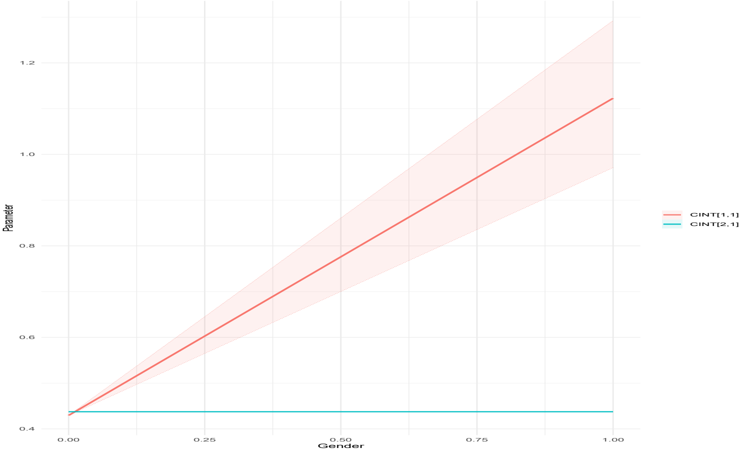
CTM Results based on long-format data

Expectations for individual parameter value change (Drift Matrix)



CTM Results based on long-format data

Expectations for individual parameter value change (Intercepts)



Discussion of the long-format results

- Using the R-package `ctsem` the long-format results of the drift matrix (Matrix A), of the autoregression and crossregression plots are not different to the wide-format results using the R-package `ctsemOMX`.
- Interpolation and extrapolation with the Kalman-Filter is possible to predict individual trajectories.
- The Stan based format models include gender as a time-independent predictor only. Models with parameter restrictions could be compared. The `ctsem` allows visualisations of the expectations for individual parameter value change for particular matrices (e.g., Matrix A) and vectors (e. g., intercepts).
- No differences of the model results using priors.

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