

Multilevel Models in Meta-Analysis: A Systematic Review of Their Application and Suggestions

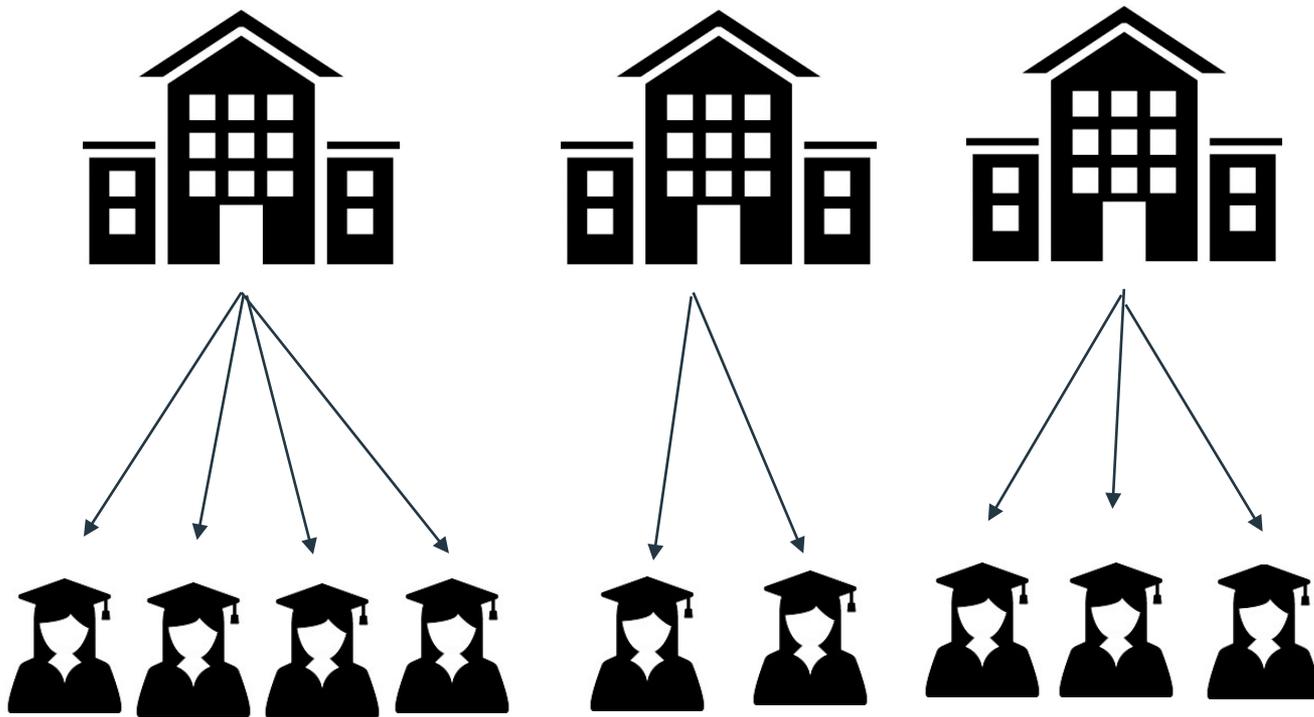
Belén Fernández-Castilla, S. Natasha Beretvas, Patrick Onghena
& Wim Van den Noortgate

KU Leuven, University of Leuven, Belgium

Research Synthesis Conference, 2019

Dubrovnik, Croatia, 31/05/19

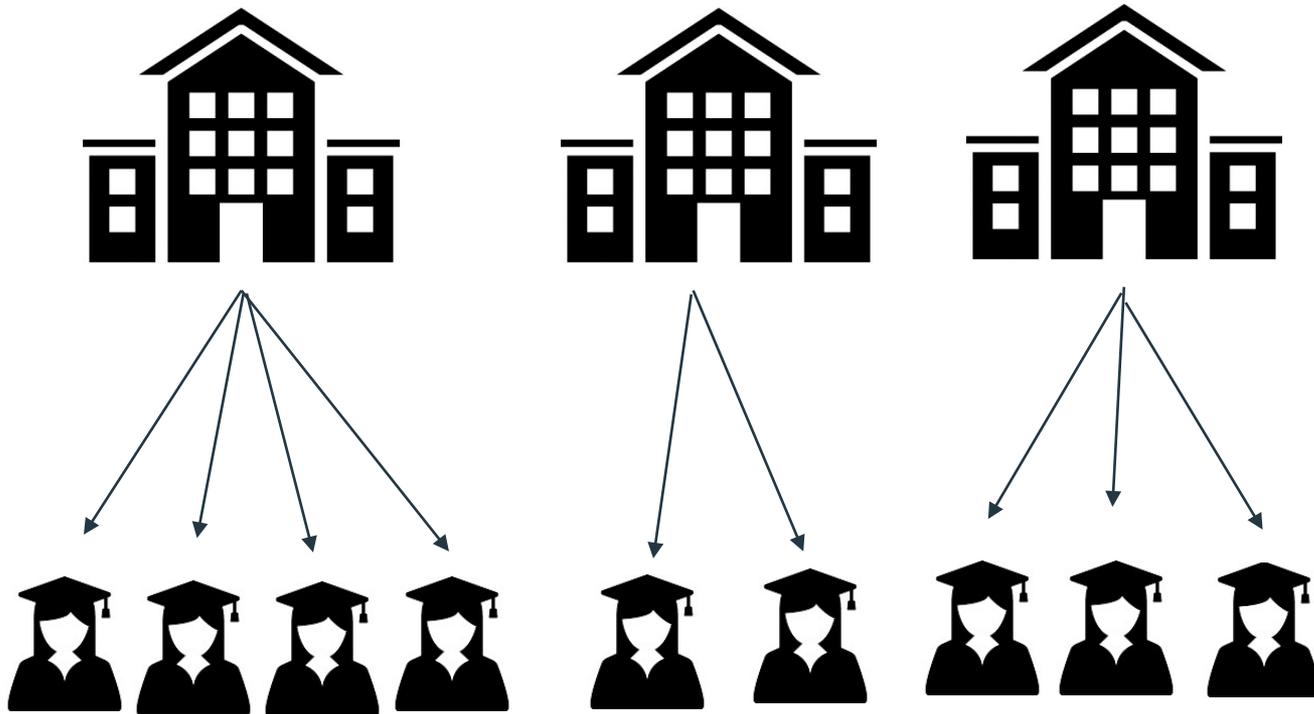
Multilevel models



School

Students

Multilevel models



School

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

Students

$$y_{ij} = \beta_{0j} + e_{ij}$$

$$y_{ij} = \gamma_{00} + u_{0j} + e_{ij}$$

Multilevel models - meta-analysis

Level 2: Studies



Study 1



Study 2



Study 3



Study 4



Level 1: Participants



Multilevel models - meta-analysis

Level 2: Studies



Study 1



ES₁



Study 2



ES₂



Study 3



ES₃



Study 4



ES₄

Level 1: Effect sizes

Multilevel models - meta-analysis

Level 2: Studies - j

Level 2: $\gamma_j = \theta_0 + u_j$



Study 1



ES₁



Study 2



ES₂



Study 3



ES₃



Study 4



ES₄

Level 1: Effect sizes

Level 1: $y_j = \gamma_j + e_j$

$$y_j = \theta_0 + e_j + u_j$$

Multilevel models - meta-analysis

Level 2: Studies - j

$$\text{Level 2: } \gamma_j = \theta_0 + u_j$$

$$u \sim N(0, \sigma_u^2)$$

Between-studies
variance



Study 1



Study 2



Study 3



Study 4



ES₁



ES₂



ES₃



ES₄

Level 1: Effect sizes

$$\text{Level 1: } y_j = \gamma_j + e_j$$

$$e \sim N(0, \sigma_{e_j}^2)$$

Sampling-variance
(assumed KNOWN)

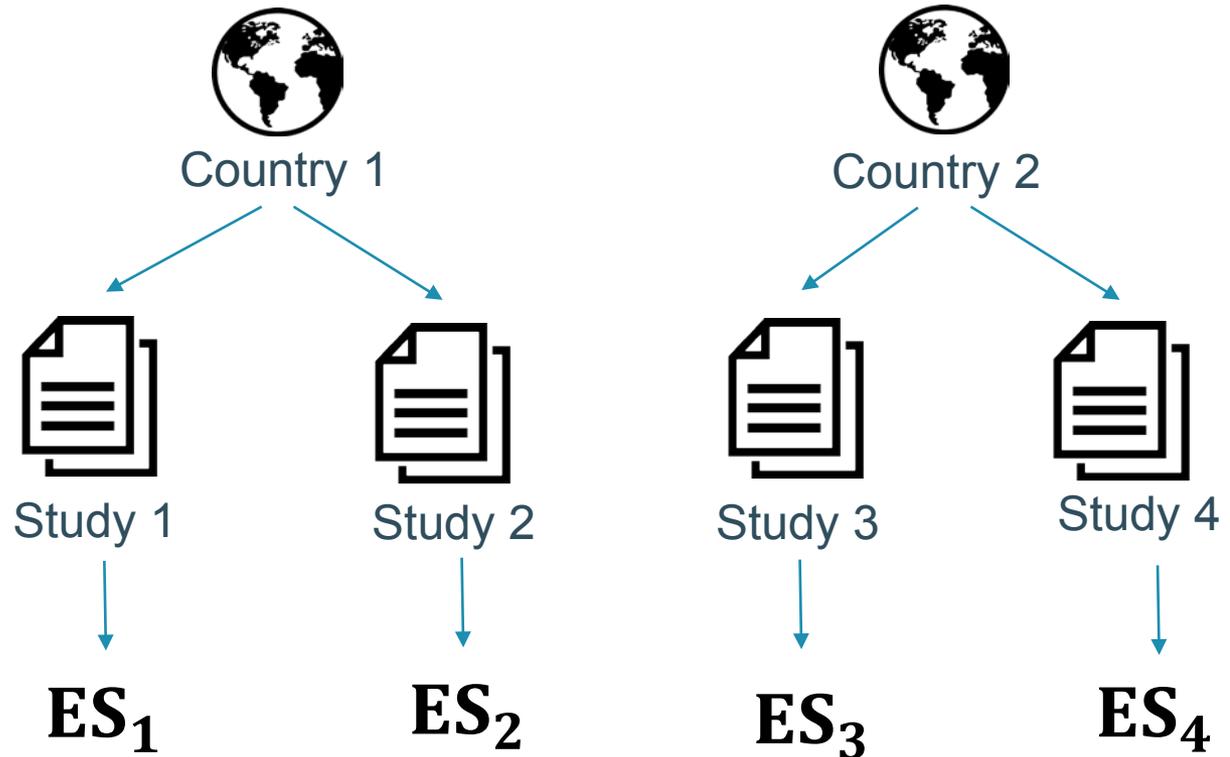
$$y_j = \theta_0 + e_j + u_j$$

Multilevel models - meta-analysis

Level 3: Countries

Level 2: Studies

Level 1: Effect sizes



Konstantopoulos (2011)

Multilevel models - meta-analysis

Level 3: Countries - k

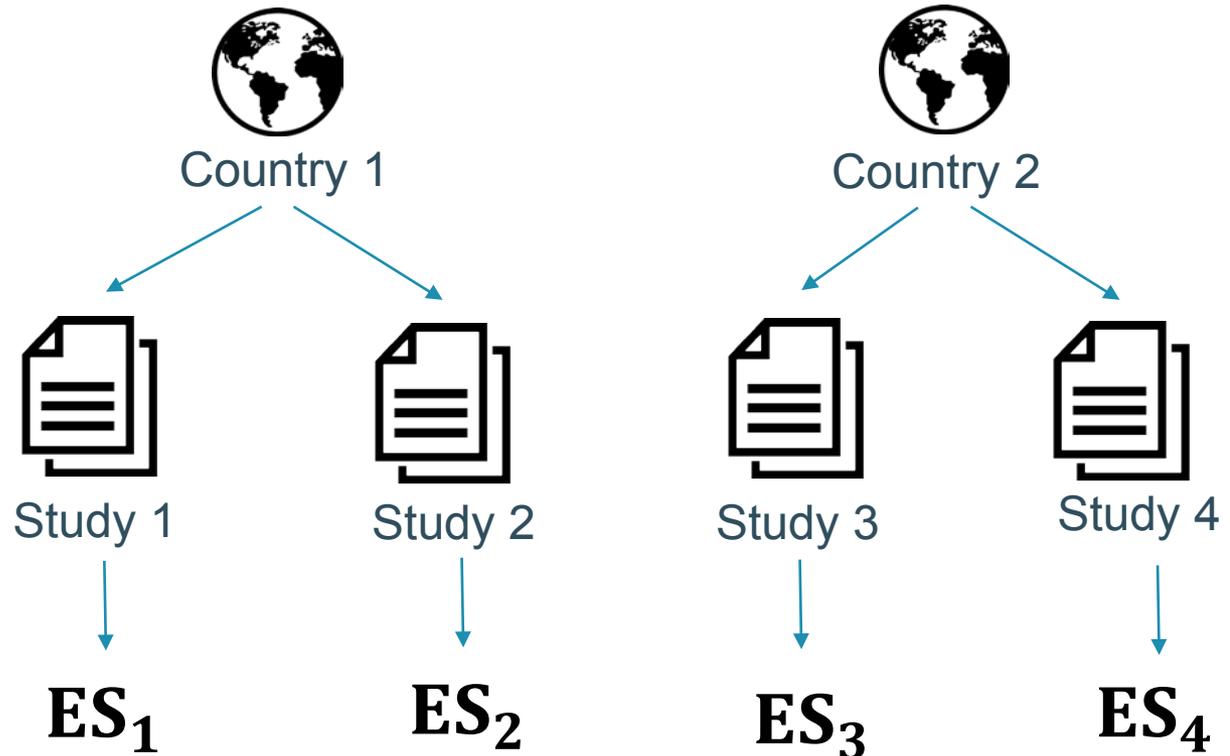
$$\text{Level 3: } \theta_{0k} = \delta_{00} + w_{0k}$$

Level 2: Studies - j

$$\text{Level 2: } \gamma_{jk} = \theta_{0k} + u_{jk}$$

Level 1: Effect sizes

$$\text{Level 1: } y_{jk} = \gamma_{jk} + e_{jk}$$



$$y_{jk} = \delta_{00} + w_{0k} + u_{jk} + e_{jk}$$

Multilevel models - meta-analysis

Level 3: Countries - k

$$\text{Level 3: } \theta_{0k} = \delta_{00} + w_{0k}$$

σ_w^2 = Between-countries variance

Level 2: Studies - j

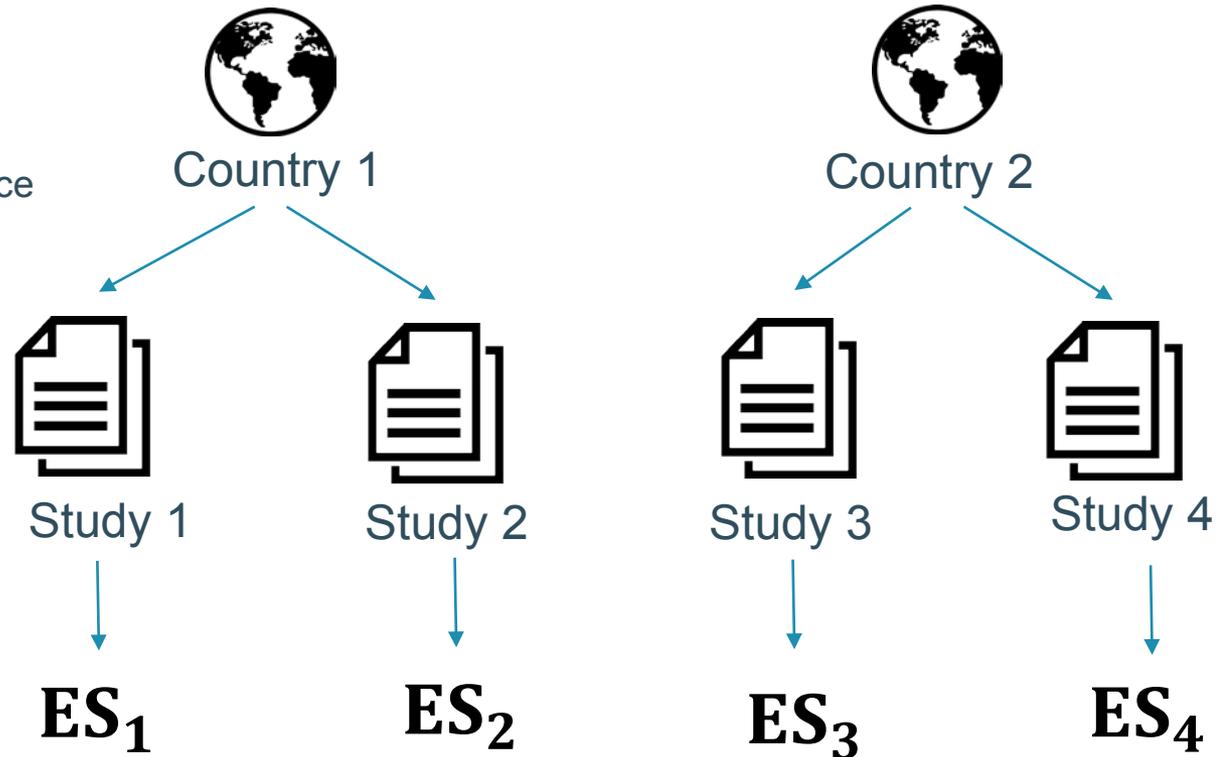
$$\text{Level 2: } \gamma_{jk} = \theta_{0k} + u_{jk}$$

σ_u^2 = Between-studies variance

Level 1: Effect sizes

$$\text{Level 1: } y_{jk} = \gamma_{jk} + e_{jk}$$

$\sigma_{e_j}^2$ = KNOWN sampling-variance



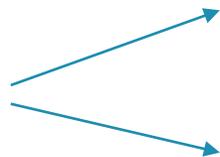
$$y_{jk} = \delta_{00} + w_{0k} + u_{jk} + e_{jk}$$

Multilevel models - meta-analysis

- Dependent effect sizes



Study 1

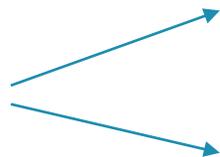


ES_{11}

ES_{21}



Study 2

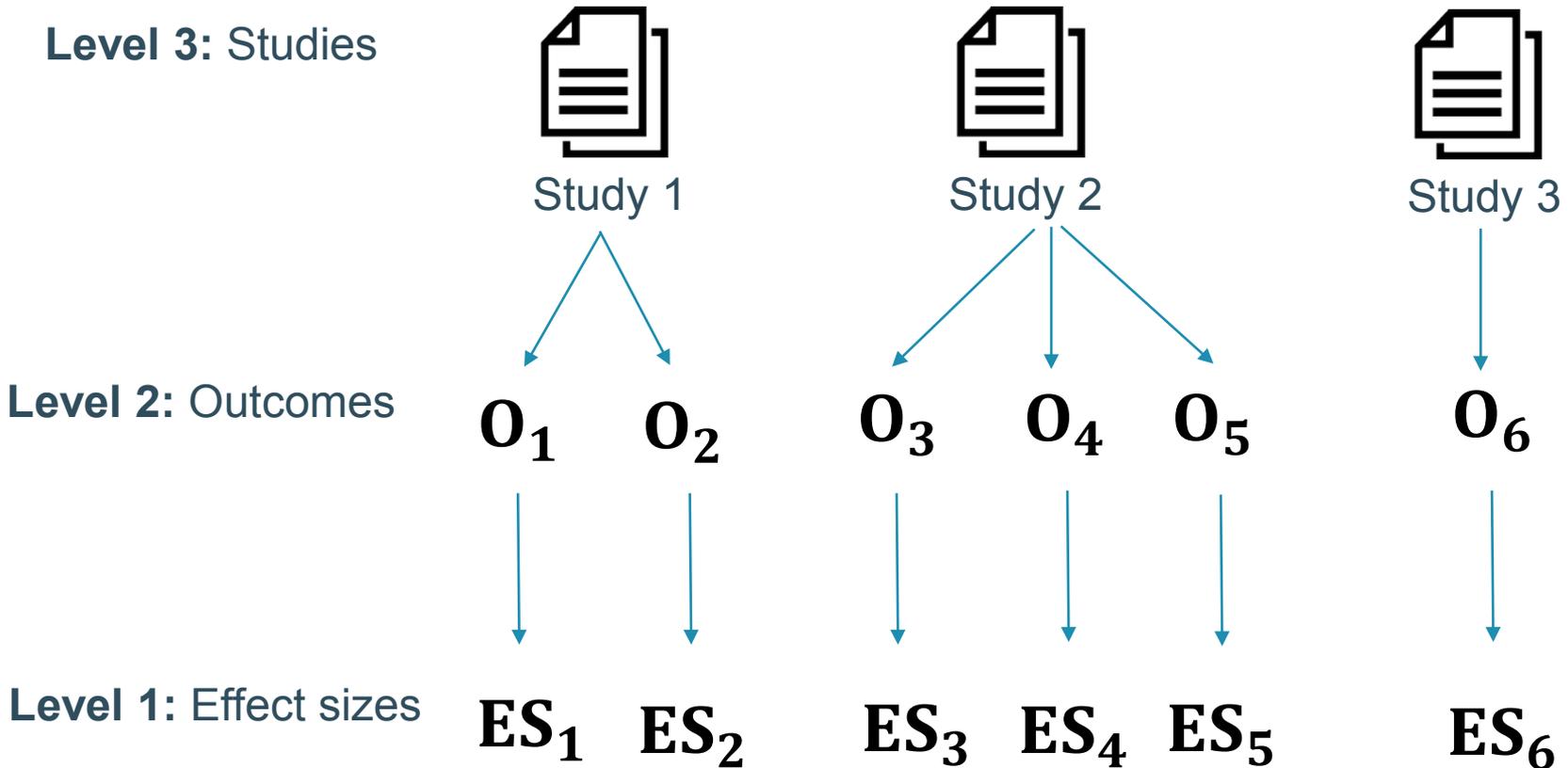


ES_{12}

ES_{22}

- Several measures of the same construct
- Several comparison between several treatment group with a common control group
- Several subsamples

Multilevel models - meta-analysis



Van den Noortgate, López-López, Marín-Martínez, & Sánchez-Meca (2013; 2015)

Multilevel models - meta-analysis

Level 3: Studies - j

Level 3: $\theta_{0j} = \delta_{00} + u_{0j}$

σ_u^2 = Between-studies variance



Study 1



Study 2



Study 3

Level 2: Outcomes - i

Level 2: $\gamma_{ij} = \theta_{0j} + v_{ij}$

σ_v^2 = Between-outcomes variance

O₁ **O₂**

O₃ **O₄** **O₅**

O₆

Level 1: Effect sizes

Level 1: $y_{ij} = \gamma_{ij} + e_{ij}$

$\sigma_{e_j}^2$ = KNOWN sampling-variance

ES₁ **ES₂**

ES₃ **ES₄** **ES₅**

ES₆

$$y_{ij} = \delta_{00} + u_{0j} + v_{ij} + e_{ij}$$

Multilevel models - meta-analysis

Level 4: Countries - k



Country 1



Country 2

Level 3: Studies - j



Study 1



Study 2



Study 3

Level 2: Outcomes - i

O_1

O_2

O_3

O_4

O_5

O_6

Level 1: Effect sizes

ES_1

ES_2

ES_3

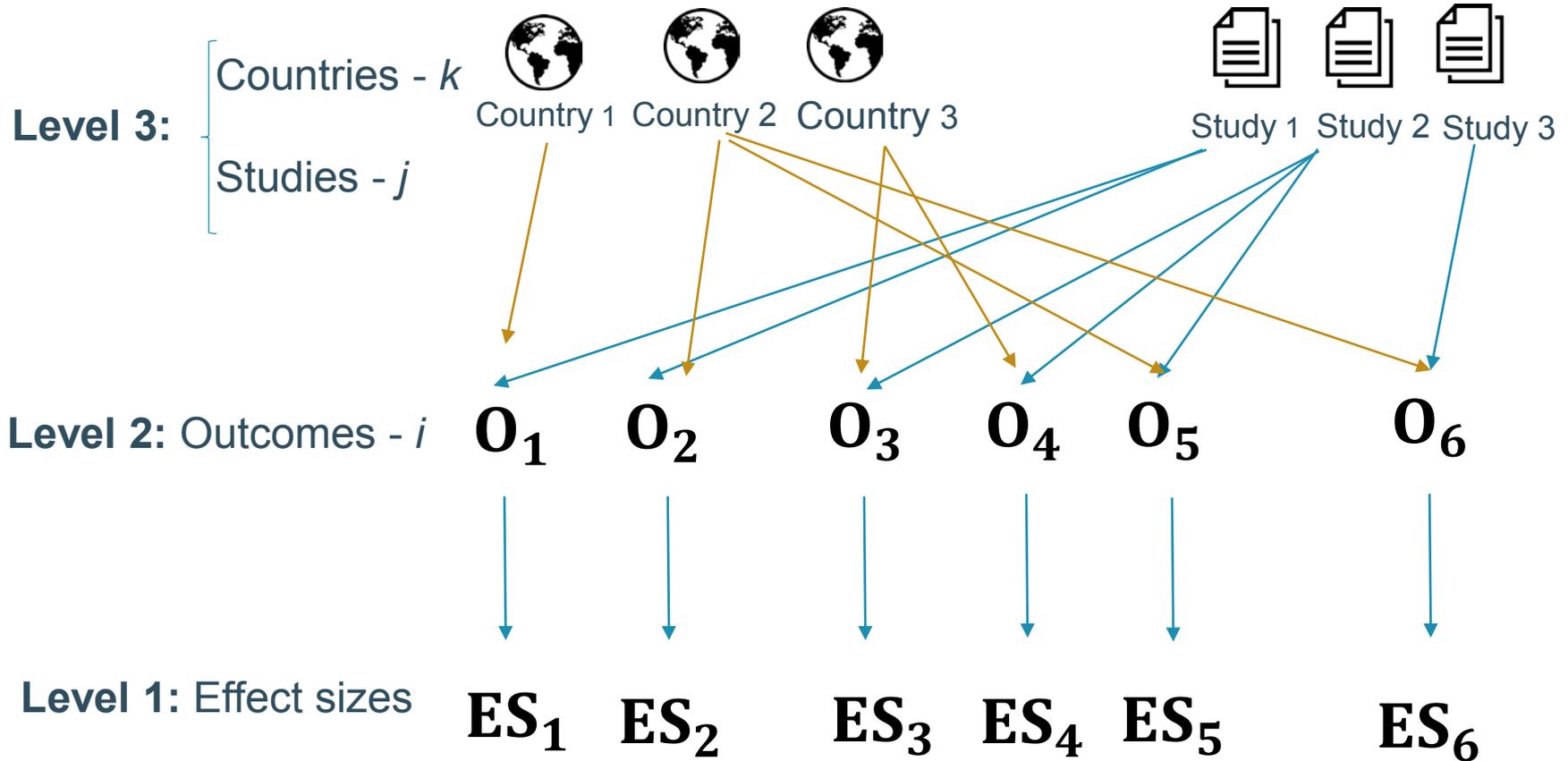
ES_4

ES_5

ES_6

$$y_{ijk} = \delta_{000} + w_{00k} + u_{0jk} + v_{ijk} + e_{ijk}$$

Other multilevel structures



$$y_{i(jk)} = \delta_{000} + w_{00k} + u_{00j} + v_{i(jk)} + e_{i(jk)}$$

Multilevel models - meta-analysis

Some advantages...

- It is possible to get the total variance disaggregated for each random effect

Some disadvantages...

- The model should include all relevant random effects

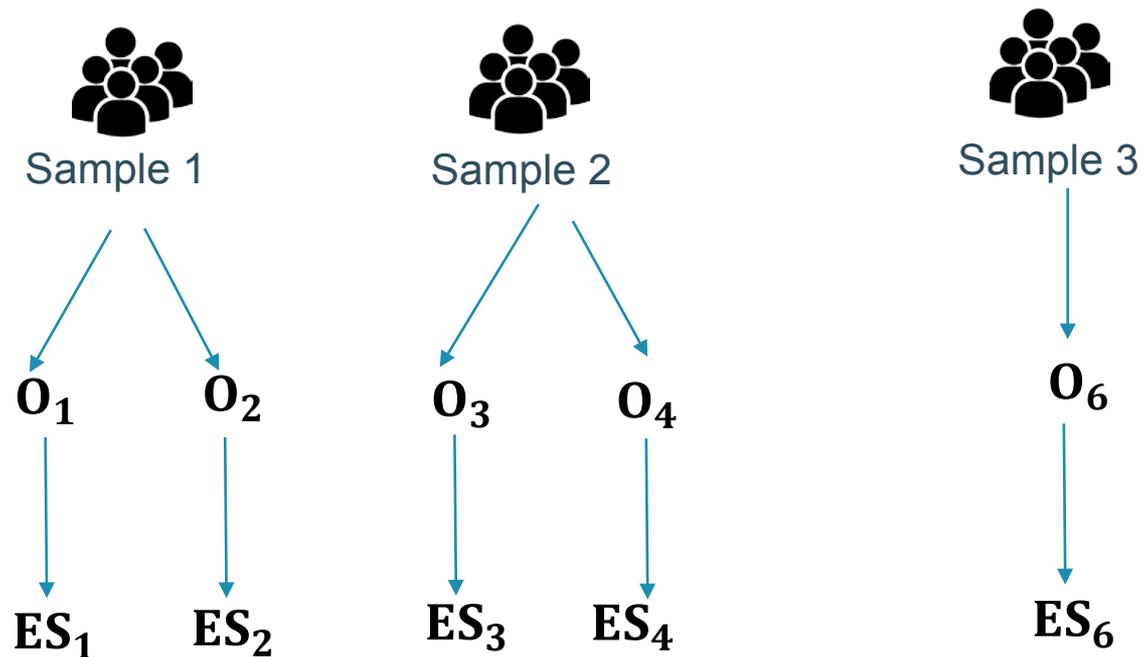
Example

Meta-analysis on the relationship between mindfulness and creativity

Level 3: Samples - s

Level 2: Outcomes - i

Level 1: Effect sizes



Example

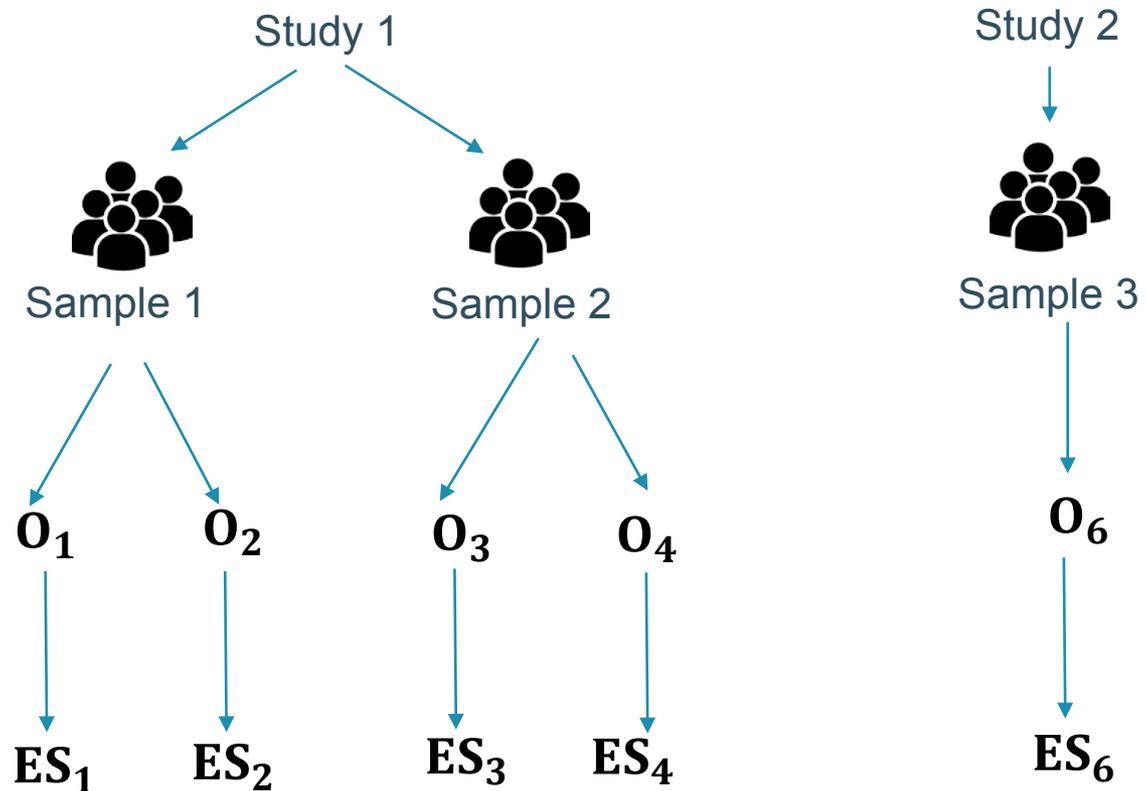
Meta-analysis on the relationship between mindfulness and creativity

Level 4: Studies - j

Level 3: Samples - s

Level 2: Outcomes - i

Level 1: Effect sizes



Example

Meta-analysis on the relationship between mindfulness and creativity

| | 3-level model | 4-level model |
|------------------------------|---------------|---------------|
| Overall effect (SE) | 0.219 (0.065) | 0.219 (0.072) |
| Level 2: between-outcomes | 0.029 | 0.029 |
| Level 3: between-samples | 0.066 | 0.054 |
| Level 4: between-studies | | 0.014 |

If model misspecified...

- Biased estimates of the...
 - Standard error of the fixed effects
 - Inflated Type I errors
 - Variance components (within-study and between-study variance)
 - Under- or over- estimated

Purpose

- How are multilevel models commonly specified in meta-analysis?
- Do researchers appropriately specify multilevel models in meta-analysis?



Systematic review

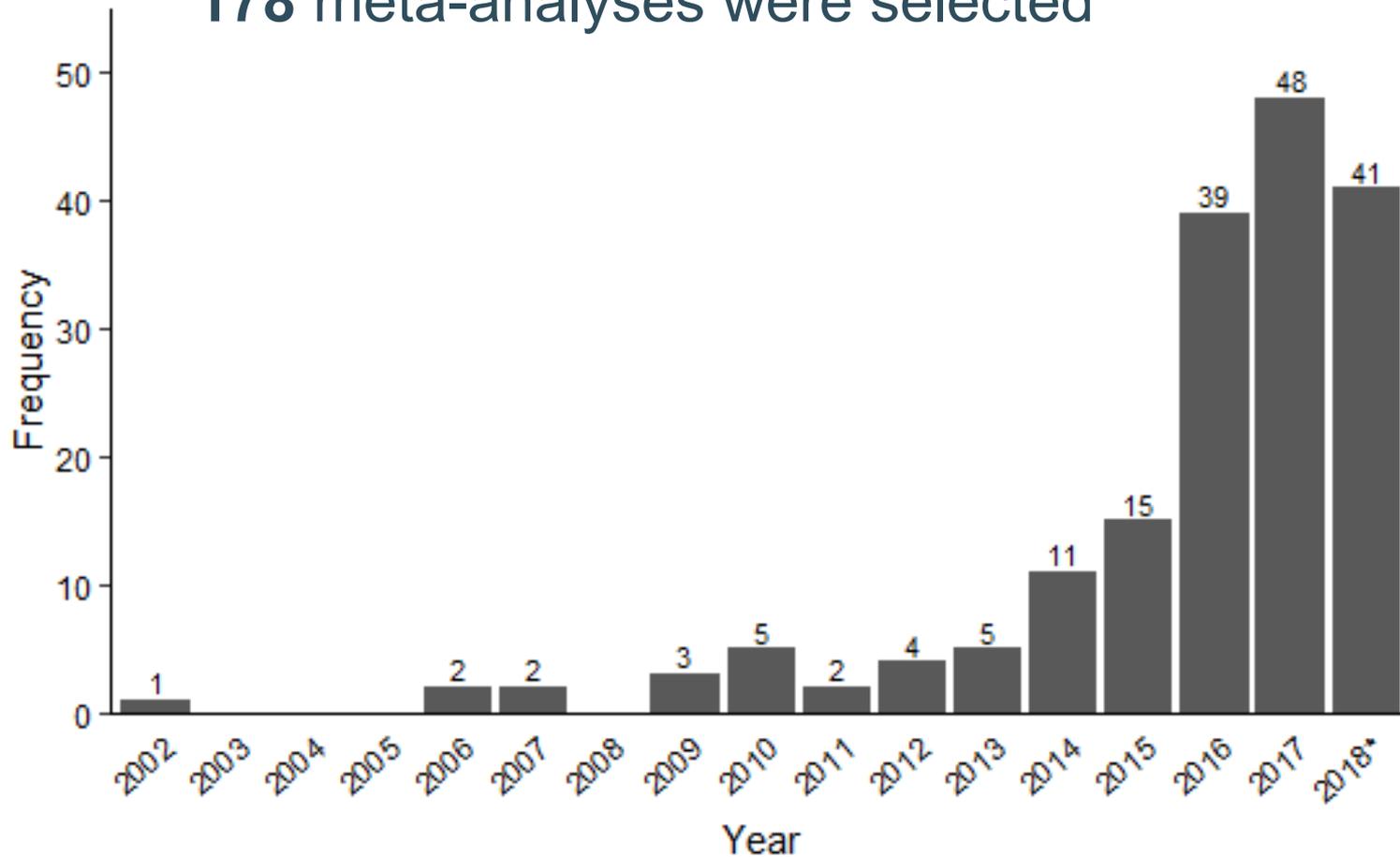
Method

Systematic search

- Meta-analysis that apply multilevel models with more than one random effect
 - Six electronic datasets: “three-level meta-analysis” OR “multilevel meta-analysis” OR “multilevel meta-analytic review”
 - Meta-analysis that cite methodological papers on the use of multilevel models in meta-analysis
 - Cheung (2014)
 - Van den Noortgate, López-López, Marín-Martínez, & Sánchez-Meca (2013; 2015)
 - Konstantopoulos (2011)
 - Raudenbush and Bryk (1985)
 - Hox and de Leeuw (2003)

Results

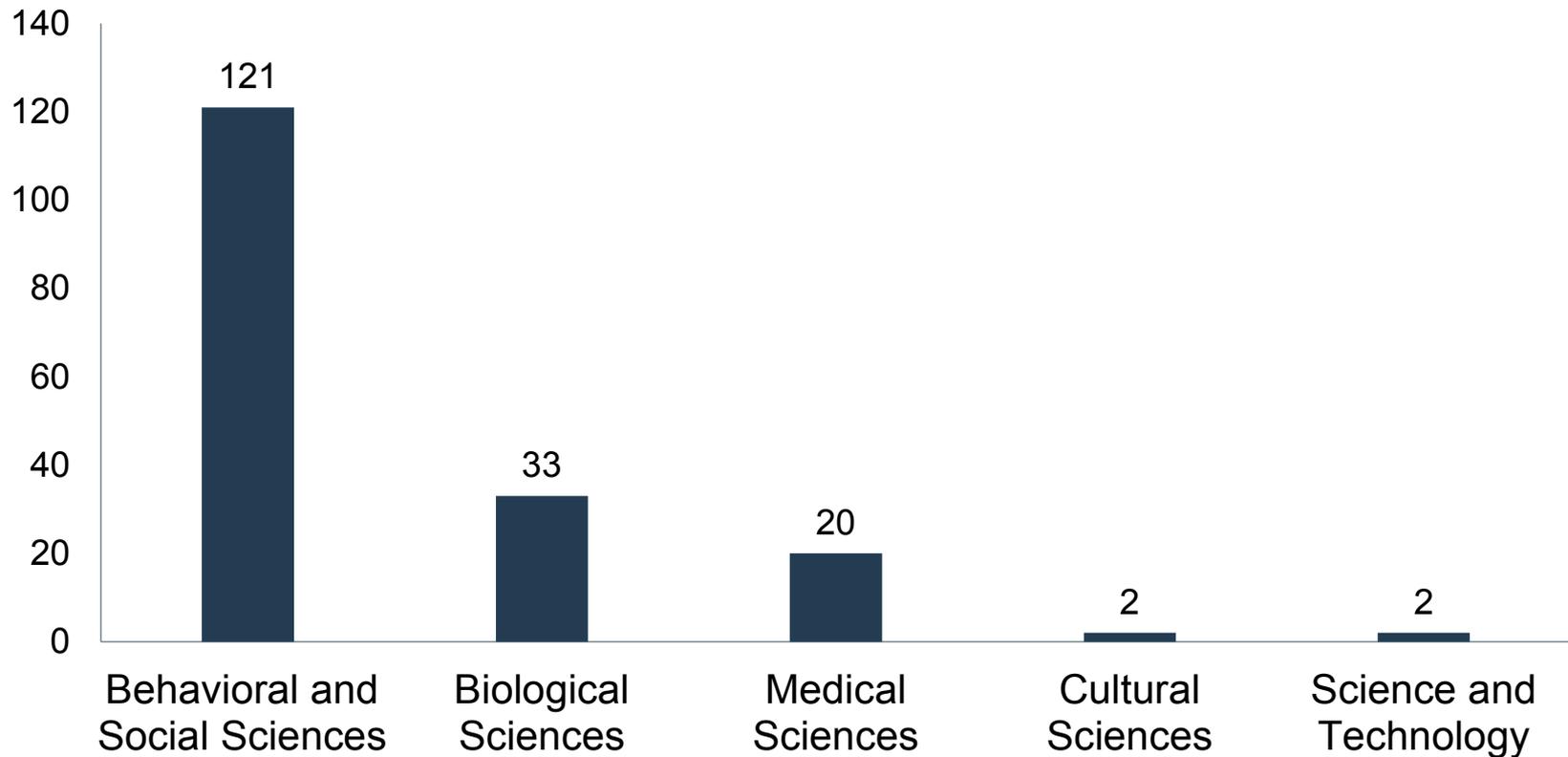
178 meta-analyses were selected



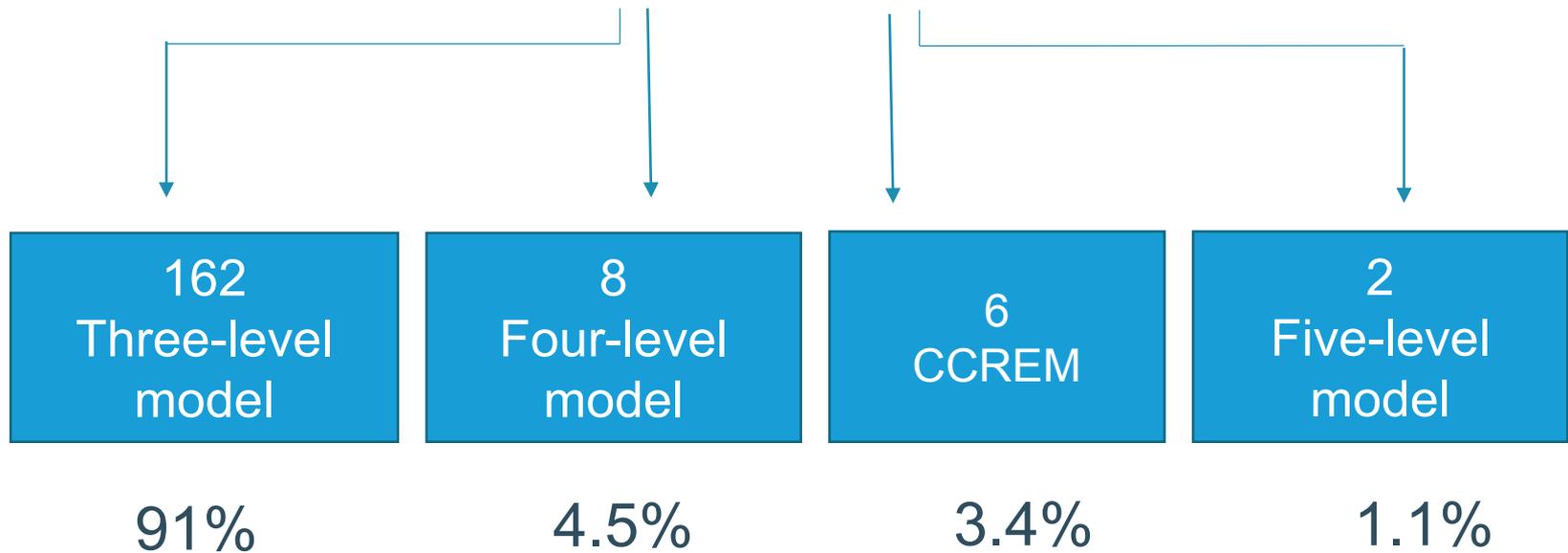
Results

178 meta-analyses were selected

Meta-analyses per discipline



178 meta-analyses



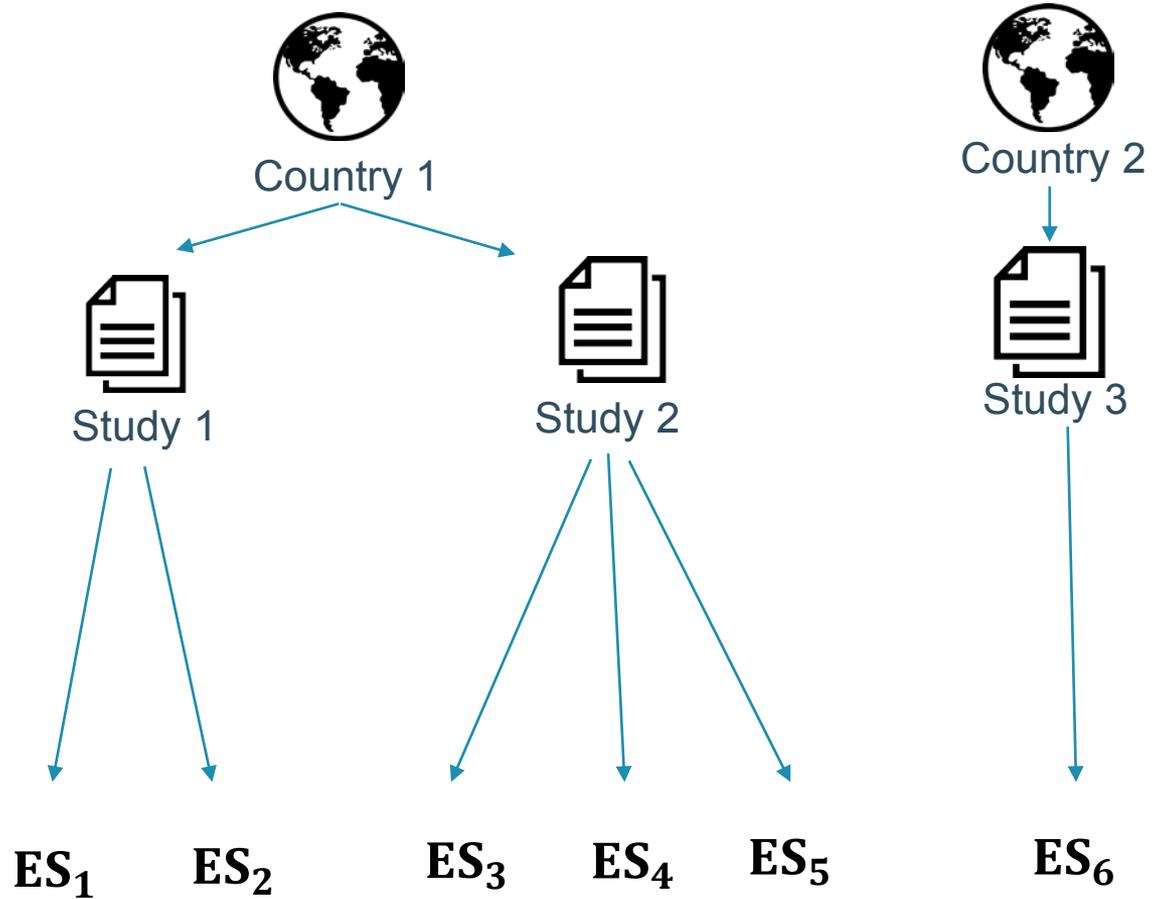
Potential alternative specifications

1. 3-level model \longrightarrow 4-level model to account for multiple outcomes within studies

Level 3: Countries - k

Level 2: Studies - j

Level 1: Effect sizes



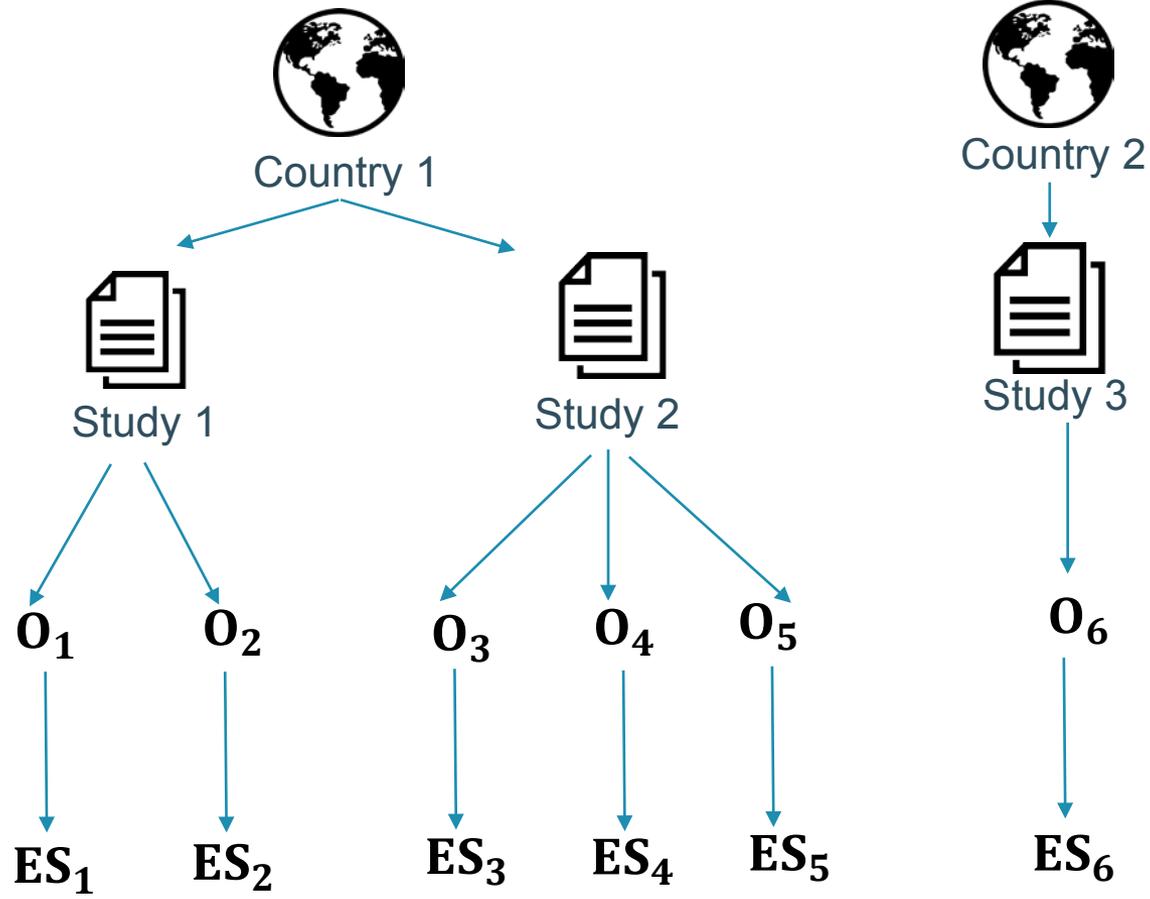
$$y_{jk} = \delta_{00} + w_{0k} + u_{jk} + e_{jk}$$

Level 4: Countries – k

Level 3: Studies – j

Level 2: Outcomes – i

Level 1: Effect sizes



$$y_{ijk} = \delta_{000} + w_{00k} + u_{0jk} + v_{ijk} + e_{ijk}$$

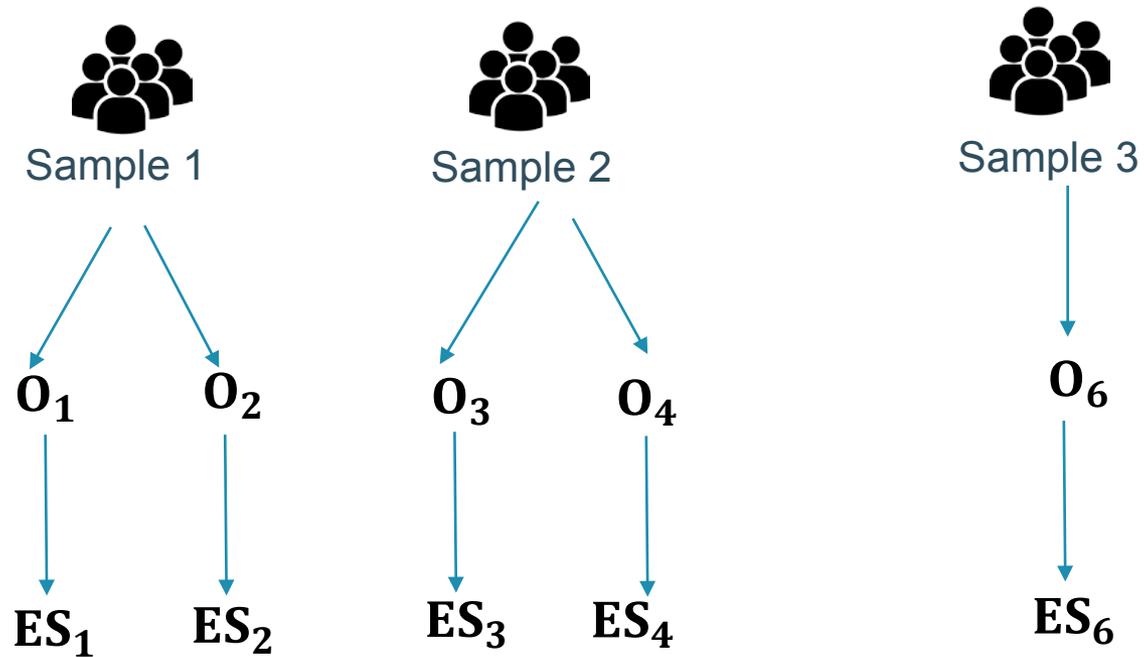
Potential alternative specifications

1. 3-level model → 4-level model to account for multiple outcomes within studies
2. 3-level model → 4-level model to account for dependency across studies.

Level 3: Samples - s

Level 2: Outcomes - i

Level 1: Effect sizes



$$y_{is} = \delta_{000} + n_{0s} + v_{is} + e_{is}$$

Level 4: Studies - j



Study 1



Study 2

Level 3: Samples - s



Sample 1



Sample 2



Sample 3

Level 2: Outcomes - i

O_1

O_2

O_3

O_4

O_6

Level 1: Effect sizes

ES_1

ES_2

ES_3

ES_4

ES_6

$$y_{isj} = \delta_{000} + u_{00j} + n_{0sj} + v_{isj} + e_{isj}$$

Potential alternative specifications

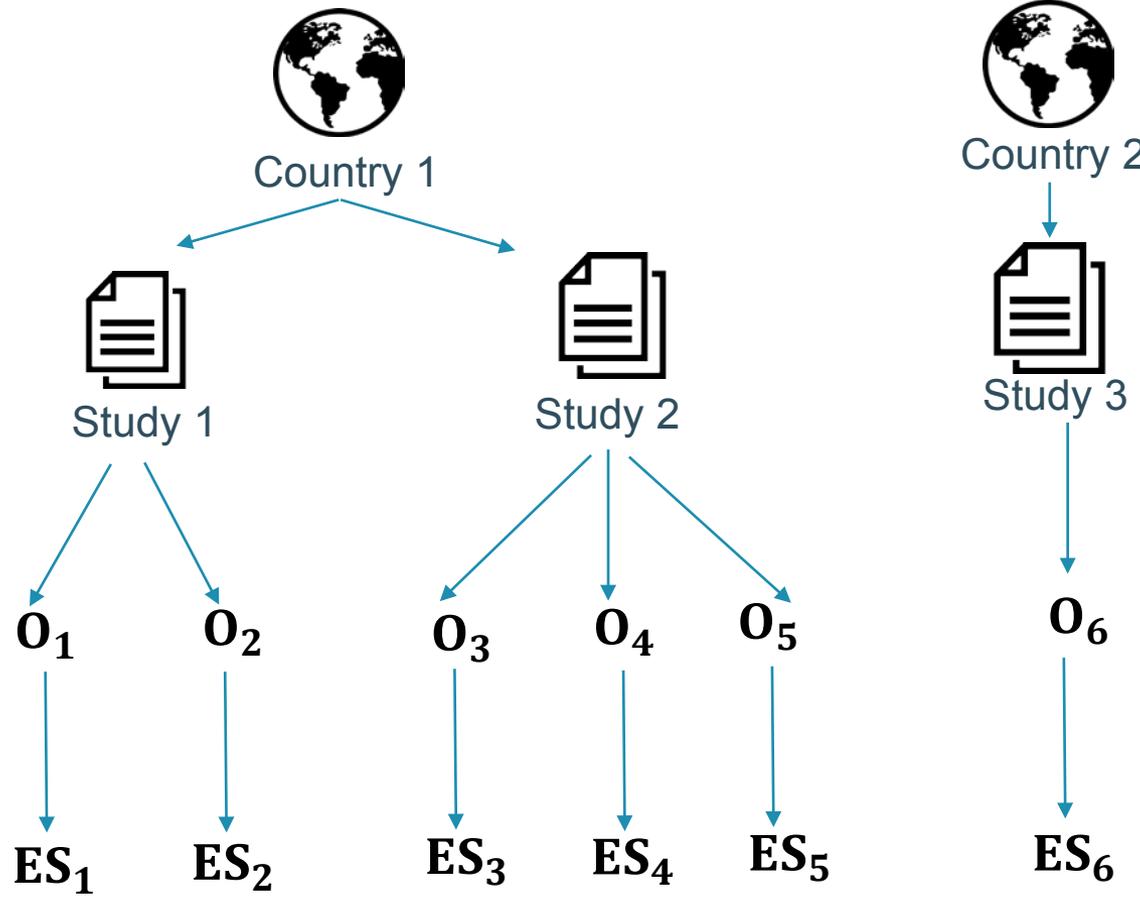
1. 3-level model \longrightarrow 4-level model to account for multiple outcomes within studies
2. 3-level model \longrightarrow 4-level model to account for dependency across studies.
3. (hierarchical) 3-level model \longrightarrow 3-level CCREM to account for crossed random factors

Level 4: Countries - k

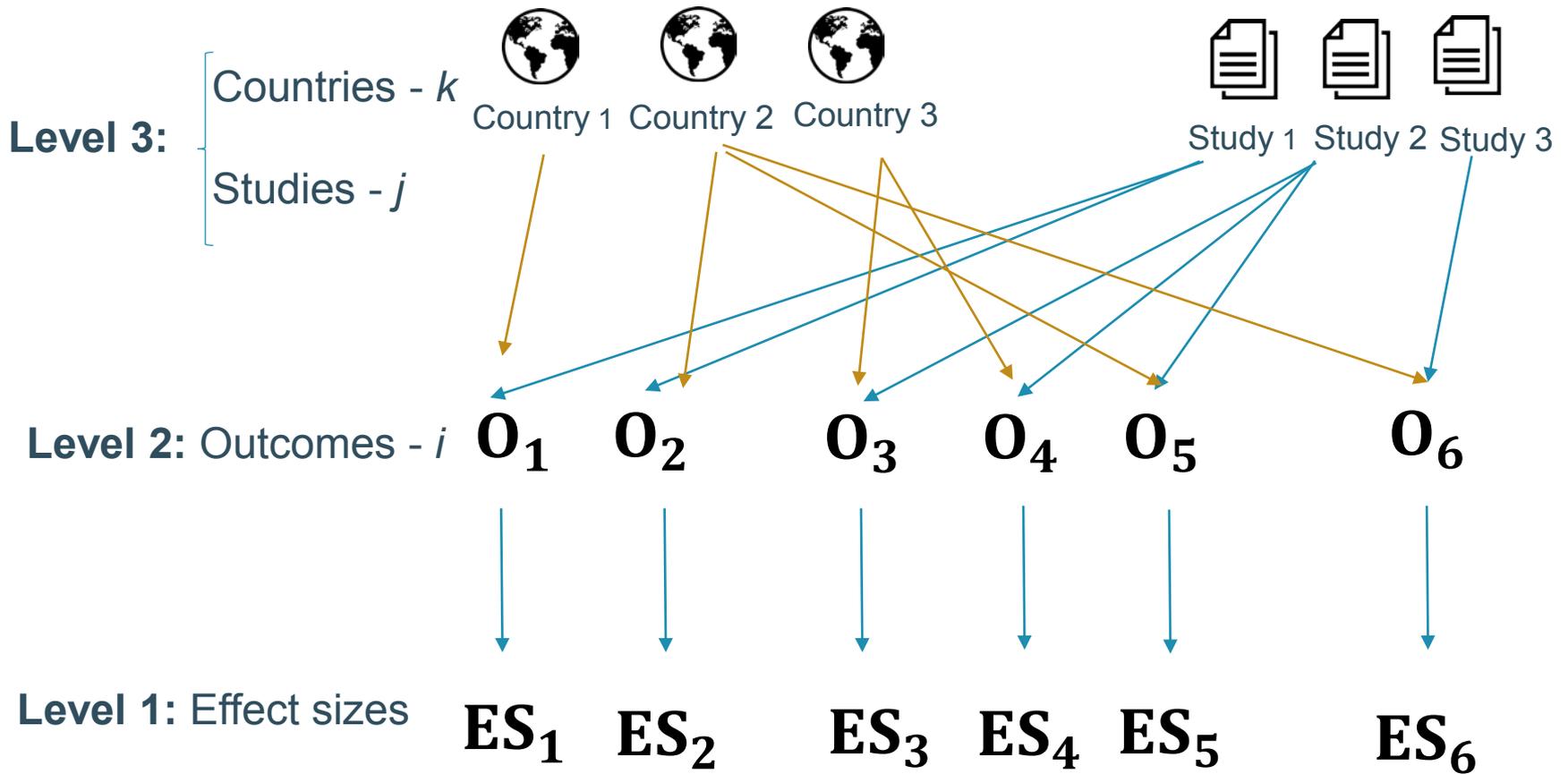
Level 3: Studies - j

Level 2: Outcomes - i

Level 1: Effect sizes



$$y_{ijk} = \delta_{000} + e_{ijk} + v_{ijk} + u_{0jk} + w_{00k}$$



$$y_{i(jk)} = \delta_{000} + e_{i(jk)} + v_{i(jk)} + u_{00j} + w_{00k}$$

Important notes

- Not everything can be treated as a random effect
- For instance: COUNTRY

| FIXED (Moderator) | RANDOM EFFECT |
|---|---|
| Few countries | Many countries |
| Interest in specific differences between some countries | Interested in variance of effect sizes due to differences between countries |

Conclusion

- Careful consideration of the data structure before performing multilevel meta-analysis
- Careful consideration of whether a variable is fixed or random
- Specify correctly the multilevel model to avoid biased parameter estimates

**Thank you very much for
your attention!**

Multilevel Models in Meta-Analysis: A Systematic Review of Their Application and Suggestions

Belén Fernández-Castilla, S. Natasha Beretvas, Patrick Onghena
& Wim Van den Noortgate

KU Leuven, University of Leuven, Belgium

Research Synthesis Conference, 2019

Dubrovnik, Croatia, 31/05/19