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Hand Position Affects Performance on Multiplication Tasks

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Abstract

We investigated whether or not hand placement affects people's ability to apply learned mathematical information in new and familiar contexts. Participants learned a set of arithmetic facts presented one way (i.e., in a × b = c format) and then were tested on those same facts shown in either a novel format (b × a = __) in Experiment 1 or in the previously-learned format (a × b = __) in Experiment 2. Throughout study and test, participants' hands were either near to or far from the stimuli. Performance on the novel format was worse when the hands were near compared to far, but performance on the previously-learned format did not depend on hand placement. Together, results indicate that hand proximity impairs mathematical performance when performance depends on the abstracting of conceptual information from sensory information. We conclude that hand placement may be involved in the application of knowledge.

Keywords: action, embodied cognition, hand placement, hand posture, STEM education, mathematical learning and performance

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Recent pedagogical advances in science, technology, engineering, and mathematics (STEM) aim to enhance student learning through hands on, interactive experiences (for recent reviews, see de Jong, Linn, & Zacharia, 2013; Freeman et al., 2014). Many instructional tools and techniques have been developed that seek to foster conceptual knowledge through manual interactions with objects that are concrete representations of to-be-learned concepts (e.g., Pouw, van Gog, & Paas, 2014). This practice has a long history in mathematics education with both researchers and educators arguing for its utility (e.g., Bruner & Kenney, 1965; Montessori, 1912). Cognitive scientists have built on this tradition by attempting to identify the cases in which such manual interactions with concrete objects can best be used in an educational setting (e.g., Laski, Jor'dan, Daoust, & Murray, 2015). Additionally, with the proliferation of mobile technologies there is now growing interest in the development of *virtual manipulatives* that allow students to interact with computer generated images that can be dynamically explored via mouse or hand movement to reveal new concepts and relationships (e.g., Moyer-Packenham & Westenskow, 2013; Ottmar, Landy, Weitnauer, & Goldstone, 2015; Sarama & Clements, 2009). An important task for psychologists, therefore, is to further clarify the degree to which, and circumstances under which, interacting with learning materials may help or hinder learning.

One recent meta-analysis of 66 studies suggests that virtual manipulatives can offer consistent benefits to learning and student achievement across many areas of mathematics and across several grade levels (Moyer-Packenham & Westenskow, 2013; see also Moyer-Packenham & Westenskow, 2016). These benefits have been linked to several possible factors, but one important aspect of virtual manipulatives seems to be the manner in which they focus and constrain attention toward the mathematical task at hand. By focusing attention on the mathematical aspects of what a problem solver is manipulating, such manipulatives can draw attention to important information while simultaneously preventing the problem solver from using the materials in less educational ways. For example, children learning about place value using a set of base-ten materials on the computer can more readily focus on how to manipulate multiple blocks to create new sets, given the ease with multiple blocks can be manipulated at the same time. In contrast, when children work with physical materials, they have the added distraction of needing to handle those multiple blocks individually and, thus, are more likely to focus on the manipulations themselves instead of on the broader idea of how these manipulations are changing place value (Sarama & Clements, 2009).

That said, benefits of virtual interactivity are not universal. Indeed, within many e-books and apps there are “hotspots” wherein interacting with the materials can draw attention away from what is being learned (de Jong & Bus, 2003; Takacs, Swart, & Bus, 2015). For example, pressing on an illustration that animates one of the characters or triggers a sound effect in an e-book can draw attention away from the story being told. The presence of these hotspots is, therefore, associated with lower levels of learning, possibly because they distract children from the to-be-learned material (Takacs et al., 2015). This same idea applies to e-learning more generally and not just interactions with e-books. In one review of 33 eye-tracking studies investigating e-learning (predominately with undergraduate populations) it was suggested that interactivity can lead to an unnecessary increase in cognitive load by forcing learners to balance their attention between what they are interacting with and the text they are reading (Yang et al., 2018).

Although the effects of interactivity with electronic formats may vary, it is clear that at least one source of its benefits and detriments is the control of attention. That is, the means by which interactivity influences attention can determine how well a student learns the given information. One factor that may influence attention during learning that has been overlooked when interpreting effects of interactivity is that of the placement of the hands. When students use manipulatives or virtual manipulatives, they not only embody actions relevant to what they are learning (e.g., grouping a set of two items from a pile of eight to “make ¼ of 8”; Martin & Schwartz, 2005), but also place their hands on or near the learning materials. How much of the observed effects can be attributed to this hand placement? Foreshadowing the current studies, we investigated the role of hand placement in people’s learning and application of arithmetic facts. Based on the extant literature, there were good reasons to suspect that hand placement would affect people’s performance in such a task, given the research highlighting the importance of both attention in mathematical cognition, and the role of hand placement in influencing attention. However, exactly how hand placement should be expected to influence performance was less clear. To begin, we first provide a brief overview of how attention influences mathematical performance and then move to summarizing the link between attention and hand placement.

Attention in Mathematical Task Performance

Research suggests that attention can influence learning and performance in STEM (see Cragg & Gilmore, 2014; Erickson, Thiessen, Godwin, Dickerson, & Fisher, 2015; Geary, 2013; Grant & Spivey, 2003; Samuels, Tournaki, Blackman, & Zilinski, 2016). It influences basic processing of numerical information, such as performance on measures of nonsymbolic numerical comparison (e.g., Fuhs & McNeil, 2013; Fuhs, Nesbitt, & O’Rear, 2018; Gilmore et al., 2013; Wilkey & Price, 2018), as well as the ability to quickly enumerate, or subitize, small sets (e.g., Olivers & Watson, 2008; Railo, Koivisto, Revonsuo, & Hannula, 2008; Vetter, Butterworth, & Bahrami, 2008). It is also important for higher-level calculation abilities, as children with higher levels of attentional control are thought to more easily use their working memory to store information about the problem they are currently solving (e.g., Swanson & Kim, 2007). Similarly, children with higher levels of attentional control show higher levels of arithmetic fluency (e.g., Fuchs et al., 2006). That is, they solve a given arithmetic problem faster than their peers with lower levels of attentional control, or they solve more problems in a set amount of time. This fluency also affects learning. As children become more fluent in their arithmetical problem solving attentional resources are freed that allow them to discover and test more advanced problem-solving strategies (e.g., Shrager & Siegler, 1998; Siegler & Araya, 2005). This may explain, in part, why children with lower levels of attentional control experience delays in advancing to more sophisticated addition strategies (Geary, Hoard, Nugent, & Bailey, 2012).

Computational modeling shows how freeing attentional resources can support better arithmetic learning (Shrager & Siegler, 1998; Siegler & Araya, 2005). Because attention exists as part of a cognitive system with limited resources, children must free up the necessary resources to be able to attend to the entire problem they are solving. Early on, a child might solve a simple addition problem (e.g., 2 + 4) by summing the two addends, starting from one (e.g., holding up two fingers then four fingers and counting 1, 2, 3, 4, 5, 6). Over time, as the child gains fluency with this procedure and fewer resources are needed to solve the problem; these extra attentional resources in turn allow the child to test out new strategies (Shrager & Siegler, 1998). Eventually, the child comes to realize that it is more efficient to solve such problems by beginning computations from the larger addend (e.g., both 4 + 2 and 2 + 4 can quickly be solved by starting at 4 and adding 2; 4…5, 6). This new strategy will become more prevalent over time given its increased efficiency. However, without gaining procedural fluency, children would not have the necessary attentional resources to shift to the new strategy that focuses first on the larger addend.

Eventually, children progress to the most efficient of arithmetic strategies—directly retrieving the answer from memory without needing to perform any computations. Here, too, attention remains important. Those with higher working memory capacity are thought to have more attentional resources available when faced with an arithmetic problem, allowing them to retrieve the answer from memory more quickly and accurately. Indeed, third and fourth grade students who can store more in their working memory are more likely to use retrieval strategies when solving addition problems (e.g., Barrouillet & Lépine, 2005).

Attentional control also plays an important role in the learning of multiplication facts. When first introduced to multiplication, learners must calculate the answer to each and every problem (e.g., 3 × 4 = 3 + 3 + 3 + 3). However, over time, the associations between specific problems (e.g., 6 × 2) and their solutions (12) become strong enough to allow the learner to simply retrieve the correct answer (e.g., Lemaire & Siegler, 1995). In other words, with experience, learners progress from a more time-intensive, error prone strategy of calculating the solution to a more accurate strategy of simply recalling the answer based on previous experience. By third grade students are already using retrieval as a common strategy for solving multiplication problems (e.g., Koshmider & Ashcraft, 1991). However, even this retrieval is not perfect.

During the process of retrieving the solution, representations for related problems can be activated (e.g., Campbell & Graham, 1985). Attentional control is thought to be key in allowing individuals to retrieve the correct response from memory by inhibiting these related but incorrect responses (e.g., Cragg & Gilmore, 2014). In other words, when viewing an arithmetic problem (6 × 3 = __), individuals will activate both the correct representation but also related representations (6 × 4 = __ or 6 + 3 = __). In order to retrieve the correct response the individual must inhibit the irrelevant representations and select the answer. This need to inhibit related representations may also explain, at least in part, why children with learning disabilities struggle with using retrieval strategies. Children with learning disabilities have lower levels of attentional control, which in turn may explain why they are more likely to respond with incorrect responses from the same multiplication table (viewing 3 × 4 and responding with the answer to 2 × 4; Barrouillet, Fayol, & Lathulière, 1997). Across these findings, it is clear that retrieval is a common strategy for solving arithmetic problems, but attention plays a key role in allowing individuals to enact such a strategy.

In addition to the role of attention, the structure of students' internally represented arithmetic facts also influences the speed with which they solve multiplication problems. Adults are thought to store two related multiplication facts in one representation (i.e., the two forms of a problem could be stored in a max × min format). For example, both 4 × 8 and 8 × 4 could be solved by accessing the same internal representation stored as 8 × 4. By storing each commutative pair within just one representation, the overall memory load of these representations is reduced (Verguts & Fias, 2005). In other words, an understanding of the principle of commutativity can be reflected in individuals' use of one representation to solve two forms of a multiplication problem. Such an understanding, however, does not mean that individuals can seamlessly access a representation no matter which way it is presented (i.e., solution times for 4 × 8 and 8 × 4 will differ). This is because individuals will need to transform a given problem to reflect their internal representation. If an individual has stored the fact in the form 8 × 4 and is asked to solve 4 × 8, they must first transform the problem to be able to retrieve the solution (Verguts & Fias, 2005). Indeed, individuals solve commutative pairs at different rates (e.g., Butterworth, Marchesini, & Girelli, 2003), though these differences in solution times do not mean that these problems are not accessing the same representation, as children as young as eight are able transfer knowledge from one learned multiplication math fact to its commutative pair (Baroody, 1999).

The summary above demonstrates how attention influences the ability to learn or process arithmetic strategies. It also shows that attention plays a role in the retrieval of multiplication facts during problem solving. Of particular relevance to the current study is the commutativity principle. In two experiments, college students studied a set of multiplication facts (of the format $a \times b = c$) and then were asked to either transfer their learning to a novel format (Exp. 1; $b \times a =$ __) or the same format (Exp. 2; $a \times b =$ __). Recall, such transfer is possible even in children (e.g., Baroody, 1999), but comes at a cost for response time (RT) (e.g., Butterworth et al., 2003). Of interest here is whether hand placement near the material will enhance or hinder such RT. In the next section, we motivate three competing hypotheses by drawing on the research on how hand placement may influence attention more broadly.

Attention, Memory, and Hand Placement

The positioning of one's hands while completing a variety of tasks can have profound impacts on performance independent of any concomitant visual and physical demands that may be associated with hand manipulation (see Abrams, Davoli, Du, Knapp, & Paull, 2008; Reed, Grubb, & Steele, 2006; Weidler & Abrams, 2013).

A variety of cognitive explanations have therefore been devised and, although the research reviewed so far should make it clear that attention is important for mathematics performance, exactly how hand placement may influence attention in such tasks is unclear.

The *enhanced attention hypothesis* supposes that attentional mechanisms are enhanced near the hands. The hypothesis stems, in part, from the fact that learning and memory are inextricably linked to attention (see Brockmole, Davoli, & Cronin, 2012, for a review). According to the enhanced attention hypothesis, people ought to perform better on a mathematics test when it is near their hands because, in many ways, they are better able to *attend* to information near their hands. For instance, in attentional cuing tasks, people are faster to react to dot-probes that happen to appear near their hand—even though they have no ostensible reason to prioritize that space over other regions of the visual field, insofar as the task is concerned (Reed et al., 2006). In the competition for attentional resources, the hands have an edge. The corollary to this, of course, is that whatever falls *beyond* near-hand space ought to be at a (relative) disadvantage—on the basis of attention being a limited resource, and there being less to go around. Using an Erikson-type flanker task, which measures how well people can maintain their focus on a central stimulus while simultaneously trying to ignore visual distractions in the immediate periphery, we found that participants were more successful (i.e., less distracted) if they had their hands (but not other barriers) positioned between the central stimulus and the peripheral distractors (Davoli & Brockmole, 2012). Taken together, these findings demonstrate that selective attention is intimately tied to near-hand space, with a particularly relevant implication for education being that people can shape their attentional window by how they position their hands.

On the other hand, the *disrupted reading hypothesis* argues that the processes that facilitate reading efficiency are reduced when the to-be-read information is placed near the hands. This hypothesis is motivated by research on the relationship between hand placement and performance on the Stroop color-word task (Davoli, Du, Montana, Garverick, & Abrams, 2010). In that study, participants exhibited a reduced Stroop interference effect when the stimuli were near to compared to far from their hands. These findings suggested that hand proximity disrupted the relative automaticity with which words were read. It is also known—through prior research using the counting Stroop paradigm (e.g., see 22222 but respond with how many digits there are, so “5”; Muroi & Macleod, 2004)—that digits, like letters and words, are read relatively automatically. If words are read less automatically near the hands, and digits are read relatively automatically like words are, then perhaps digits would also be read less automatically near the hands. That is, the readable text of a multiplication problem (like 17 × 5 = __) should be processed more slowly when it is near the hands, leading to longer RT.

The above hypotheses predict that hand proximity should either enhance or impair performance on symbolic arithmetic problems. Although contradictory in their predictions, those hypotheses are unified by fact that they do not take into account possible differences in performance across specific contexts or circumstances: They respectively argue that hand proximity is beneficial and should be generally exploited, or that hand proximity is detrimental and should be generally avoided. A third possibility, of course, is that the answer is more nuanced. Under this view, whether hand proximity enhances or impairs (or does nothing to) performance depends on the characteristics of the stimuli at study and test.

The primary evidence in support of the context-dependent view comes from research conducted in the domain of visual learning. In one study (Davoli, Brockmole, & Goujon, 2012), participants viewed hundreds of images of complex, vibrant geometric imagery, like fractals and kaleidoscopic patterns, and searched for a target letter embedded within each image. While the task itself did not explicitly call for learning, learning was nevertheless possible. Over the course of the study, several of the geometric patterns repeated, and these repeated patterns were always predictive of target location. Thus, by learning the relationship between patterns and target locations, observers stood to decrease their search times. Importantly, in one variant of the study the patterns maintained their original color scheme with each repetition, whereas in another variant they appeared in a brand new color scheme each time. Learning the statistical association between pattern and target location in spite of a changing color scheme required relatively flexible perceptual criteria—essentially, being able to recognize an old pattern in new colors as being "something old" (i.e., previously experienced) and not “something new.”

When the patterns were repeated in the exact same color scheme as before, hand proximity had no effect on participants’ ability to learn the association between shape and target location. That is, learning was just as good near the hands as it was farther away. However, when the repeated shapes appeared in a brand new color scheme with each presentation, a striking reduction in participants’ ability to learn the shape-target association near their hands was observed. Together, these results show that the effect of hand proximity on visual learning was not universal but context-dependent. More specifically, the context in which hand proximity impaired learning was the one that required participants to relax their criteria about what constituted an “old” image. This implies that hand proximity impairs the ability to abstract information from, and recognize valid transformations of, sensory data—giving rise to the *impaired abstraction hypothesis*. According to this account, hand proximity ought to impair math performance insofar as performance depends on recognizing valid transformations of previously learned information. While it may appear that the Davoli, Brockmole, and Goujon (2012) study already challenges the enhanced attention and disrupted reading hypotheses, Davoli et al. focused on the incidental learning of relatively meaningless visual patterns in a task that did not demand learning to take place. As such, it is not clear that those findings would generalize to scenarios where more meaningful materials are used in an explicit learning task, as in the current study, which focused on the visual representation of mathematical concepts.

The Current Study

Like objects in the physical world, mathematics concepts are rarely bound to a single, unchanging sensory representation. Consider how learning arithmetic includes learning to recognize that 7 × 12 = 84 and 12 × 7 = 84 describe the same mathematical fact (see Verguts & Fias, 2005 for a review of how such internal representations could be stored). At all stages of mathematical development, conceptual fluency involves learning which perceptual differences are to be tolerated across exemplars and which ought not to be. Far from being trivial, these perceptual factors are being revealed to play a much larger role in mathematical learning and performance than previously thought (see Kellman & Massey, 2013, for a review). In a particularly striking demonstration of this relationship, algebra students who had been trained to recognize valid transformations of algebraic equations exhibited dramatic improvements in their problem-solving speed, going from nearly 30 s per problem before training to 12 s per problem after training (Kellman, Massey, & Son, 2010). Thus, it is not a strictly academic point that mathematics performance involves the abstraction of sensory information, and our focus on this feature of mathematics in the present study is rooted in its theoretical as well as practical relevance.

To determine the effect hand placement has on mathematics performance, we adapted a paradigm that has been used for studying learning and transfer of arithmetic facts (Chesney & McNeil, 2014). In our version, undergraduate participants learned a set of multiplication facts presented one way (i.e., in $a \times b = c$ format) and then were tested on those same facts shown in either a novel format ($b \times a =$ __) in Experiment 1 or in the previously-learned format ($a \times b =$ __) in Experiment 2. Some participants completed the task with their hands near the information during study and test, and others completed the task with their hands farther away (see Figure 1). It is important to note here that in this paradigm, undergraduates receive time to first study the set of multiplication facts and then receive a practice test allowing them to attempt to answer the problem before seeing the correct answer. Given this extended introduction and practice with these multiplication facts, the current studies are meant to measure individuals’ ability to retrieve these facts from memory, rather than testing individuals’ ability to perform the necessary computations.

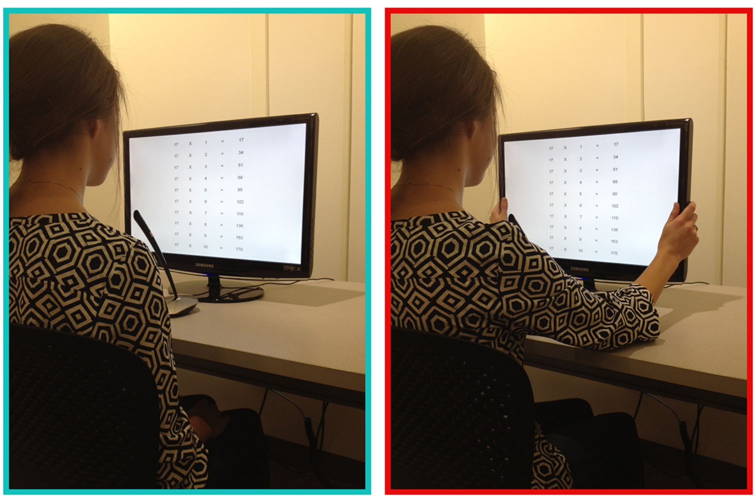

Figure 1. The hands far (left) and hands near (right) postures.

Note. For display purposes, this figure shows the 17’s set onscreen with the microphone positioned on the table. During the actual study, however, the microphone was not positioned until prior to the final test.

This paradigm allowed us to distinguish among our three hypotheses. If the enhanced attention hypothesis is correct, then we should observe better performance near the hands on both test formats (Exps. 1 and 2), because hand-proximity should better allow participants to focus attention on the task. If the disrupted reading hypothesis is correct, then we should observe worse performance near the hands on both test formats (Exps. 1 and 2), because hand-proximity should reduce the relative automaticity with which the stimuli in the task would be read. If the impaired abstraction hypothesis is correct, then we should observe worse performance near the hands on the novel format (Exp. 1), and no difference in performance between the postures on the previously-learned format (Exp. 2), because hand-proximity should impair the abstraction of conceptual information from sensory information, thus making it difficult to recognize valid transformations of previously-learned multiplication facts.

Experiment 1

In Experiment 1, participants learned a set of mathematical facts based on multiples of 17. At study, these facts were presented in $a \times b = c$ format (e.g., 17 × 5 = 85). At test, participants had to solve these same problems in $b \times a =$ __ format (e.g., 5 × 17 = __). This structural switch in format from study to test constituted a valid transformation of the same mathematical concept despite changes to the sensory information.

Method

Participants

Our target number of participants was 72 (36 per posture). This required our recruiting 75 participants total, as data from three had to be excluded due to noncompliance with instructions. Participants were undergraduates at the University of Notre Dame, were experimentally naive, and participated in exchange for course credit. All participants provided informed consent, and the study was approved by the institutional review board at the University of Notre Dame.

Apparatus and Display

Testing was done on a desktop computer in a sound-attenuated testing room. Stimulus presentation and data collection were controlled by Experiment Builder software (SR Research). All stimuli were presented on a 22-inch LCD computer monitor in black font against a white background. Participants' vocal responses (see below) were collected via microphone, and an integrated voice key trigger in Experiment Builder was used to detect the onset of the voiced response. Each response was recorded to a .wav file.

Task

The main objective of the task was for participants to learn 10 math facts based on the 17's multiplication table from 1 to 10 (henceforth, the *17's set*). The task consisted of three main phases: a study phase, a practice test, and a final test.

During the study phase, participants were given 5 minutes to study a table that contained the 17's multiplication set. All facts were displayed on the same screen, and each fact appeared in $a \times b = c$ format. Participants were instructed to study these facts for a subsequent test.

The practice test presented the 17's set one problem at a time in random order, six times total (i.e., six blocks of 10). Each problem appeared in $a \times b =$ __ format (i.e., without its answer). After 5 seconds, the full math fact appeared (i.e., $a \times b = c$) and remained onscreen for another 2 seconds. The purpose of this phase was for participants to practice generating the correct answer to each problem on their own in preparation for the upcoming test. To that end, participants were instructed to speak their answers aloud during this phase—ideally, prior to the correct answer appearing onscreen. Data were not collected during this phase, however. The correct answer was always shown after each problem so that participants could check their accuracy in real time.

The final test presented the 17's set one problem at a time in random order, three times total (i.e., three blocks of 10). Each problem appeared in $b \times a =$ __ format, and participants had a maximum of 10 seconds to respond with their answer. Participants spoke their responses into the microphone. A blank screen replaced the problem immediately upon response (i.e., through triggering the voice key) or once the time limit was reached. Unlike the practice test, the correct answer was never shown during the final test. The next problem appeared 2 seconds after the onset of the blank screen.

Procedure

After providing informed consent, participants were brought into the testing room and seated at the computer. Participants then read through a set of instructions that described the three phases of the task (see above) as well as how to use the microphone to respond. In particular, it was emphasized that participants should speak directly into the microphone, loudly and clearly, while also avoiding filled pauses and other extraneous noises.

Next, the experimenter positioned participants into their assigned hand posture. Participants in the *hands far* posture rested their hands in their laps, while participants in the *hands near* posture positioned their hands at either side of the monitor (Figure 1). Participants were instructed to maintain their hand posture throughout the three phases of the task, and participants' hands were monitored via video camera to ensure compliance with those instructions.

Once participants were in their assigned posture and ready to proceed, the experimenter left the room and the study phase began. After 5 minutes of study, the practice test began, with an onscreen reminder to participants that they should speak their answers aloud during this phase. Following the practice test, participants were given the opportunity to take a break, during which they were free to move from their assigned posture. After this, the experimenter came into the room and positioned the microphone at a set location on the table directly in front of participants. Participants were reminded how to respond using the microphone, readopted their hand posture, and were left by the experimenter to complete the final test.

Design

Block (1, 2, or 3) refers to the first, second, or third set of 10 problems during the final test. All participants completed all three testing blocks. Hand posture (hands near or hands far) was manipulated between-subjects and assigned on an alternating basis. Together, these yielded a 3 (Block: 1, 2, or 3) × 2 (hand posture: hands near or hands far) mixed-factorial design.

Dependent Variables

Our primary variable of interest was RT, measured in milliseconds. RT was defined as the time between the appearance of a test problem and the onset of a vocal response, and it was measured for each problem of the final test (i.e., 30 problems in all). We also scored the accuracy of each response on the final test by going back through the individual .wav files and comparing the recorded response against the correct answer. In order for a response to be scored as accurate, it had to be the correct answer and delivered all at once. Responses with pauses or stalls in the middle (e.g., "Onnnnnnne………nineteen" to the problem 7 × 17 = __) were not considered accurate, as such behaviors indicated that participants had begun their response prior to having accessed the answer, which would invalidate their RT. If a response contained multiple answers, only the first one was considered. So, for instance, a response of "83—no, 85!" to the problem 5 × 17 = __ would not be scored as accurate.

Results

The data for Experiment 1 are included in the supplementary materials. Trials that did not meet the criteria for an accurate response (see above) were excluded from further analysis (3.8%). Correct trials with RT ≥ 6,000 ms (1.3%) were considered inattention errors and were also excluded from analysis. Thus, the overall accuracy rate for Experiment 1 was 95.1%.

Figure 2 shows the mean RTs at each block for each hand posture. A 3 (Block: 1, 2, or 3) × 2 (hand posture: hands near or hands far) mixed-model analysis of variance (ANOVA) confirmed that RT decreased with block, $F(2, 140) = 27.1$, $p < .001$, $\eta_p^2 = .279$. The more times participants saw a problem, the faster they were to respond with the answer, indicating that participants gained fluency with the math facts with repeated testing. Importantly, the ANOVA also confirmed that responses were slower overall in the hands-near condition compared to the hands-far condition, $F(1, 70) = 4.39$, $p = .04$, $\eta_p^2 = .059$. Hand posture and block did not interact, $F(2, 140) = 1.88$, $p = .16$, $\eta_p^2 = .026$, indicating that the relative cost of the hands-near condition to RT remained stable across testing (Block 1: 14.03% cost; Block 2: 13.81% cost; Block 3: 9.53% cost).

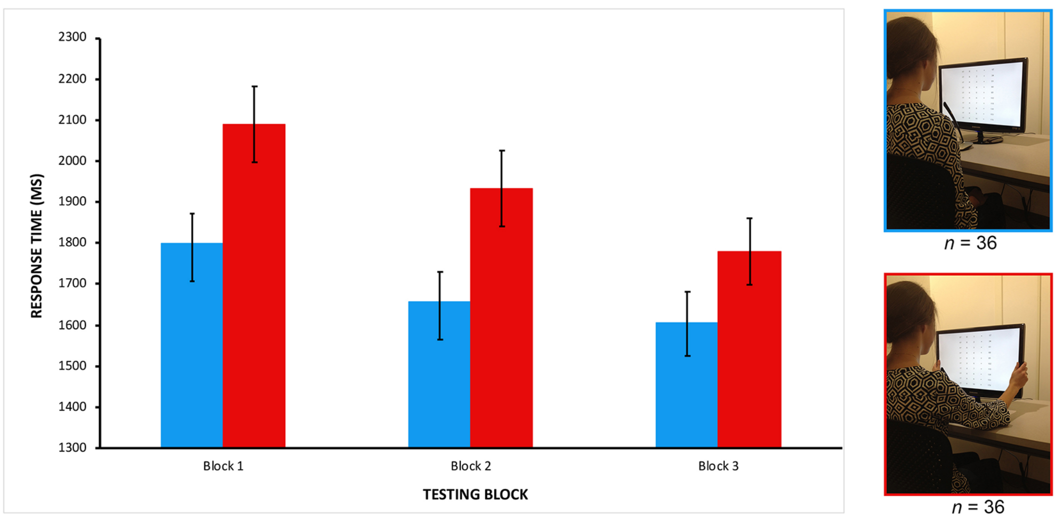

Figure 2. Mean response times at each block in the hands-far (blue) and hands-near (red) postures in Experiment 1.

Note. Error bars show the standard error of the mean.

Table 1 (left) shows the mean number of problems solved correctly per block for each hand posture.

Table 1

Descriptive Statistics for the Number of Problems Solved Correctly for Experiments 1 and 2

	Experiment 1				**Experiment 2**			
	Hands Far		**Hands Near**		**Hands Far**		**Hands Near**	
Block	***M***	***SD***	***M***	***SD***	***M***	***SD***	***M***	***SD***
1	9.28	0.88	9.47	0.84	9.33	0.86	9.19	1.17
2	9.61	0.69	9.61	0.73	9.75	0.44	9.50	0.97
3	9.53	0.65	9.56	0.73	9.81	0.47	9.56	0.77

Accuracy data were submitted to a 3 (Block: 1, 2, or 3) × 2 (hand posture: hands near or hands far) mixed-model ANOVA. In general, accuracy differed from block to block, increasing from Block 1 to 2 and decreasing slightly from Block 2 to 3, but this effect was not statistically significant, $F(2, 140) = 2.92$, $p = .06$, .040. Critically, accuracy did not differ across hand postures, $F(1, 70) < 1$, and hand posture and block did not interact, $F(2, 140) < 1$. On the whole, these findings confirm that the differences observed in RTs (above) are not attributable to condition-specific speed-accuracy tradeoffs.

Discussion

The main finding from Experiment 1 was that participants who completed the study with their hands near the stimuli performed *worse* than those whose hands were farther away, as revealed through differences in RT. These data do not support the enhanced attention hypothesis, which had predicted that performance should have been better near the hands. Instead, these results provide tentative support for both the disrupted reading hypothesis and the impaired abstraction hypothesis, both of which had predicted that performance should have been worse near the hands—although, for different reasons. Thus, it is not clear from the Experiment 1 results alone whether performance suffered near the hands because the relative automaticity with which the digits were read was reduced or because the ability to abstract conceptual information from sensory information, and thus recognize valid transformations of learned concepts, was impaired. In Experiment 2 we collected additional data to help distinguish between these two possibilities.

Experiment 2

In Experiment 1, the sensory information changed from study to test such that performance depended on participants' ability to apply their knowledge of previously-learned math facts to new instances (i.e., valid transformations). Critically, the disrupted reading hypothesis predicts worse performance (i.e., slowed responses) near the hands regardless of whether the sensory information at test was the same as or different from what was learned during study. This is based on our understanding that digits will be read with relative automaticity, even if they have been seen before. Indeed, we know this from the Stroop task. If prior exposure to readable sensory information was enough to reduce the relative automaticity with which that information would be read on any subsequent appearance, then incongruent stimuli in the Stroop task (e.g., WHITE in the color-word paradigm; 22222 in the counting paradigm) would only cause interference the first time they were shown; but this is known to not be the case. Meanwhile, the impaired abstraction hypothesis predicts worse performance near the hands only when the sensory information changes from study to test, but not when it stays the same. This is because the latter does not require the participant to engage in any further conceptual abstraction of the sensory information at test; they merely need to the match a test item to a previously stored instance.

In order to distinguish those possibilities, then, we modified our mathematical learning paradigm from Experiment 1 so that performance no longer hinged on abstraction of sensory information. In terms of design, the only difference between Experiments 1 and 2 was the format of the test problems relative to what was studied. At study, the math facts were again presented in $a \times b = c$ format. At test, participants had to solve these problems in $a \times b =$ __ format—that is, the same format in which the facts had been learned. Under these conditions, the disrupted reading hypothesis predicts that performance should still be worse when the hands are near compared to far, while the impaired abstraction hypothesis predicts that performance should not differ between postures.

Method

Participants

Our target number of participants was again 72 (36 per posture). This required our recruiting 74 new participants total, as data from two had to be excluded due to noncompliance with instructions. Participants were undergraduates at the University of Notre Dame, were experimentally naive, and participated in exchange for course credit. All participants provided informed consent, and the study was approved by the institutional review board at the University of Notre Dame.

All other details of the method were identical to Experiment 1, with one exception: The problems during the final test appeared in $a \times b =$ __ format.

Results

The data for Experiment 2 are included in the supplementary materials. Our analysis of the Experiment 2 data was identical to that of Experiment 1 unless otherwise stated. Trials that did not meet the criteria for an accurate response (see above) were excluded from further analysis (3.3%). Correct trials with RT ≥ 6,000 ms (1.4%) were considered inattention errors and were also excluded from analysis. Thus, the overall accuracy rate for Experiment 1 was 95.3%.

Mean RTs are shown in Figure 3. An ANOVA confirmed that RT decreased with block, $F(2, 140) = 11.1$, $p < .001$, $\eta_p^2 = .137$, suggesting that, as in Experiment 1, participants gained fluency with the math facts with repeated testing. Critically, however, and unlike in Experiment 1, RT did not differ between hand postures, $F(1, 70) < 1$. In fact, responses were numerically *faster* in the hands-near posture than in the hands-far posture (though not statistically so), reflecting an apparent reversal of what was found in Experiment 1 (we statistically evaluate the differences between Experiments 1 and 2 below). Hand posture and block did not interact, $F(2, 140) < 1$.

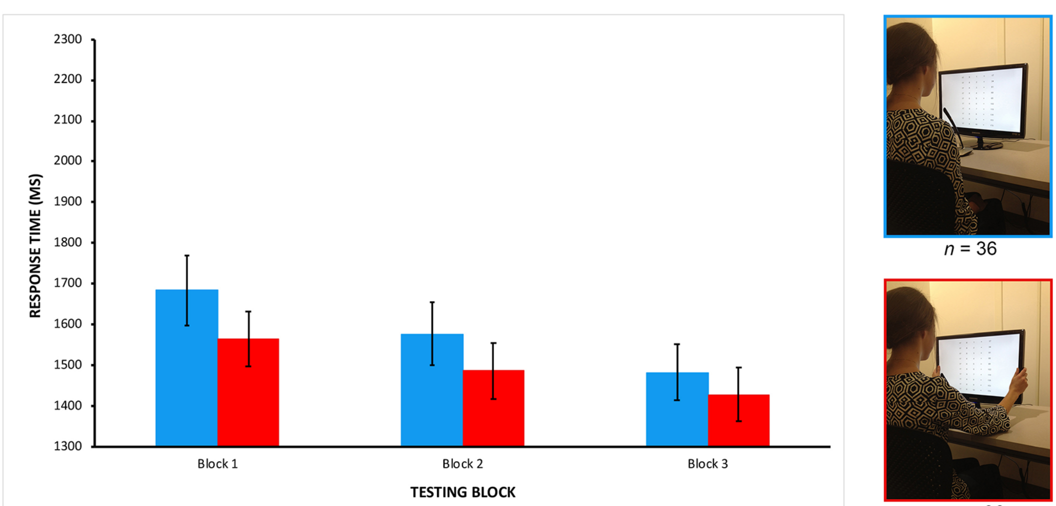

Figure 3. Mean response times at each block in the hands-far (blue) and hands-near (red) postures in Experiment 2. Error bars show the standard error of the mean to facilitate comparison between groups.

Mean accuracy data are shown in Table 1 (right). Accuracy data were submitted to an ANOVA. The ANOVA revealed a main effect of block, $F(2, 140) = 5.06$, $p < .01$, $\eta_p^2 = .067$. In general, accuracy increased from Block 1 to 2 to 3. Importantly, as in Experiment 1, accuracy did not differ across hand postures, $F(1, 70) = 2.91$, $p = .092$, $\eta_p^2 = .040$, and hand posture and block did not interact, $F(2, 140) < 1$.

Between-Experiment Analysis

In order to more directly compare the critical findings of Experiments 1 and 2, we conducted a 3 (Block: 1, 2, or 3) × 2 (hand posture: hands-near or hands-far) × 2 (Experiment: 1 or 2) mixed-model ANOVA on RT. The ANOVA showed that RT decreased with block, $F(2, 280) = 35.3$, $p < .001$, $\eta_p^2 = .201$, as would be expected based on the earlier results. RTs were faster in Experiment 2 than in Experiment 1, $F(1, 140) = 14.3$, $p < .001$, $\eta_p^2 = .093$, indicating that participants performed better when tested on items in a previously-learned format compared to a novel format. There was no main effect of hand posture, $F(1, 140) = 1.28$, $p = .260$, $\eta_p^2 = .009$. Most importantly, hand posture interacted with experiment, $F(1, 140) = 5.07$, $p = .026$, $\eta_p^2 = .035$. Planned pairwise comparisons were conducted to clarify the nature of the interaction. Independent samples *t*-tests showed that, for the hands-near condition, RT was significantly slower in Experiment 1 than in Experiment 2, $t(70) = 4.07$, $p < .001$, while for the hands-far condition, RTs did not differ between experiments, $t(70) = 1.14$, $p = .26$. The significant difference for the hands-near effect between the experiments remains significant when using the Bonferroni correction (.05 divided by the two follow-up tests = .025). Overall, these analyses confirm on a statistical level that performance depended on the interaction between hand position and the stimulus characteristics of the task. No other interactions (i.e., those involving block) were significant, *F*s < 1.75, *p*s > .18, $\eta_p^2 = .013$.

Accuracy data were also submitted to a between-experiment ANOVA. As would be expected based on the earlier results, accuracy generally increased with block, $F(2, 280) = 12.0$, $p < .001$, $\eta_p^2 = .079$. No other main effects or interactions were significant, *F*s < 1.88, *p*s > .17, $\eta_p^2 < .014$.

Discussion

The critical finding from Experiment 2 was that we did not find evidence that RT differed between the hands-near and hands-far postures. These data do not support the disrupted reading hypothesis, which predicted that performance should have been worse near the hands in this experiment—just as it was in Experiment 1—due to a reduction in the relative automaticity with which stimuli would be read. On the other hand, these data support the impaired abstraction hypothesis, which predicted that the hands-near and hands-far postures would yield similar RTs in this experiment, because here—unlike in Experiment 1—the stimuli at test did not require participants to engage in any further conceptual abstraction of sensory information. We elaborate on these findings below in the General Discussion.

General Discussion

In the present study we examined the role of hand placement in math performance. Participants learned a set of math facts presented one way ($a \times b = c$) and then were tested on those same facts in either a novel format ($b \times a =$ __) in Experiment 1 or in the previously-learned format ($a \times b =$ __) in Experiment 2. Our main finding was that hand proximity impaired performance at test when the format changed from study to test, but we did not find evidence of the same impairment when the format stayed the same. From these results, we can conclude that hand proximity does not seem to universally enhance or impair mathematics performance; instead, hand proximity impairs mathematics performance under testing conditions that rely on the abstraction of common conceptual information from valid transformations of sensory information.

Before discussing the theoretical implications of our findings, it is important that we address a potential alternative explanation—namely, that it was simply more difficult to respond in the hands-near posture, perhaps owing to the unnaturalness of that posture compared to having the hands in the lap. Based on that explanation, the slowing we observed near the hands in Experiment 1 would have had nothing to do with differences in how people perceive and thus learn information near or far from their hands, but rather would have been an artifact of our methodology. However, two aspects of our data are incompatible with that alternative. First, RTs did not differ between hand postures in Experiment 2. Although this may have been because it was so easy for individuals to transfer their learned multiplication facts to the same format that they studied (further discussed below), responses were numerically *faster* near the hands. If merely adopting the hands-near posture had made responding more difficult, then we should have observed slower responses in that posture in Experiment 2, as well. Second, if the hands-near posture had made it more difficult to respond, then we perhaps should have seen concomitant reductions in accuracy in that posture. However, error rates did not differ between the postures in either experiment. We thus think that our results reflect actual differences in visual processing near to versus far from the hands.

Could factors other than hand *placement* account for our findings? From a historical perspective, it seems unlikely. Across the field's 10+ years of activity, research has routinely and repeatedly addressed—and ruled out—potential confounds that come along with manipulations of hand placement. Careful experimentation has demonstrated that near-hand effects cannot be explained by concomitant changes in hand visibility, visual feedback, tactile feedback, arm posture, postural comfort, response modality, response characteristics (e.g., direction of the response when pushing buttons), whether or not the hands are occupied, and numerous other factors (for representative examples, see Abrams et al., 2008; Reed et al., 2006; Weidler & Abrams, 2013). Moreover, near-hand effects have been reported by researchers from around the world using a host of behavioral, neuropsychological, and electrophysiological techniques (e.g., see Tseng & Davoli, 2015). Based on this burgeoning research literature, we think it within reason to conclude that our effects are based on the changes in visual processing that accompany changes in hand placement.

How does the present study build on our theoretical understanding of hand-altered visual processing? The context-dependent nature of our main finding perhaps holds a clue. Recall that we did not find evidence that hand proximity disrupted the application of mathematical knowledge when abstraction of sensory information *was not* required, but we *did* when abstraction *was* required. That asymmetry could be interpreted as a form of inflexibility, a resistance to engage in alternate (or, perhaps, subsequent) forms of analysis, and it is remarkably similar to other forms of apparent inflexibility near the hands. For instance, people take longer to initiate switches between global and local scopes of analysis of objects near their hands—a finding that suggests a sluggish form of volitional attention (Davoli, Brockmole, Du, & Abrams, 2012). Likewise, near their hands, people are slower to shift their attention from one object to another across space (as in visual search) and across time (as in rapid serial visual presentation)—effects which, collectively, are thought to reflect delayed disengagement of attention (Abrams et al., 2008; Davoli & Abrams, 2009; Vatterott & Vecera, 2013). Taking those findings together with the present study, it appears as though hand proximity may engage a mode of visual processing that "locks in" to the sensory information at hand, while having the hands farther away enables a more flexible mode of processing. Indeed, another way to view the current findings is that the delayed responding in Experiment 1 was the result of individuals being "locked in" on the unfamiliar format, which needed to be transformed to fit the representation that had been studied. In other words, having hands near the material made it harder for individuals to re-organize a given problem (e.g., 3 × 17) to fit their abstract representation for the math fact (e.g., 17 × 3). Since such a shift of attention is not necessary in the familiar test condition, having hands near the material did not influence performance.

It is worth noting at this point that the null findings in Experiment 2 may also reflect an interaction between generally increased attention near the hands as well as a disrupted reading for materials near the hand. That is, the factors behind the enhanced attention hypothesis and the disrupted reading hypothesis may not be all-or-nothing, but rather are relative effects that are both influencing performance at a given time. If this is the case then the null result may reflect a wash between the negative influence of disruptive reading with the positive influence of enhanced attention. That being said, future research will need to investigate this effect more closely, as it is unclear whether this is truly a null effect or if we were underpowered in the current study to find this effect. The smaller difference between conditions in Experiment 2 may reflect the fact that individuals were nearing their capacity for their speed in retrieving the multiplication facts, making it harder to detect any advantages that placing hands near the material may bring. Future research can begin to get at this by using more complex problems, thereby increasing the overall time it generally takes individuals to process and retrieve the relevant solution.

On the one hand, our findings are in line with prior research showing that hand placement affects other forms of visual learning (e.g., recognizing patterns in complex geometric imagery; Davoli, Brockmole, & Goujon, 2012). On the other hand, these results are counterintuitive when it comes to expectations about the role that the hands ought to play in STEM education, particularly with respect to so-called "hands-on learning" techniques. In short, hands-on learning is an interactive pedagogical tool that aims to enhance student learning through first-hand, multisensory experiences. While hands-on learning aims to build students' conceptual understanding by increasing their physical engagement with to-be-learned material, it is clear from the present study that students' ability to *apply* their conceptual knowledge in novel circumstances may be better served by keeping their hands farther away.[i] From a translational perspective, then, the present line of research on hand placement opens the door for new inquiries into how broadly—or how literally—a hands-on approach ought to be applied in STEM education. This is not to say that the current study is immediately applicable to findings on interactive learning more broadly. Rather, the current studies provide important findings about one aspect of such learning, hand placement, that is often overlooked. With the rise of mobile technology that brings learning closer to the hands, such effects should not be ignored. Future research will need to take into consideration not only how manipulating or interacting with the learning materials may influence learning, but also how the proximity to the hands influences learning.

It is important to acknowledge a limitation of our study, namely, that we cannot be sure whether the effects of hand placement occurred during study, during test, or during both study and test. In other words, did hand placement affect learning, performance, or both? Because our measures occurred during the test phase, our data reflect only the extent to which participants were able to apply learned information during the test. Future research should address the questions of whether and how hand placement affects learning versus performance by systematically varying hand placement at study and at test.

It is also important to acknowledge that the current paradigm served only as a test of the influence of hand placement on individuals' ability to retrieve recently learned multiplication facts in different formats, and not a measure of mathematical processing (or learning) more broadly. This is not to say that the current findings do not have applications for mathematical education; the current results show how hand placement can influence the efficiency of the most commonly used solution strategy, retrieval (e.g., Lemaire & Siegler, 1995). Future research will need to elucidate just how these findings might transfer to the classroom. One area that may be particularly relevant involves the use of computers in learning. Multiplication fact learning is often accomplished using flashcards and with paper-and-pencil exercises. Both of these activities, however, require the learner to study material while it is near to the hands. Computers, however, allow the learner to view and manipulate information further from the hands. Of course, applications of these findings could be directed to means of teaching that overcome the cognitive constraints associated with hand placement rather than interventions that modify the learner's body. Studies that replicate and manipulate our findings using realistic classroom scenarios would increase the ecological validity of these effects.

Although attention is involved in many areas of mathematical cognition (Cragg & Gilmore, 2014; Geary, 2013), how the findings translate to these areas remains to be seen. For example, how hand placement may influence *computation*, as opposed to retrieval, may be different. If a given calculation would benefit from using a previously learned strategy in a new context (e.g., decomposition, or breaking a more complex problem down into simpler problems; Siegler, 1987), hand placement may hinder performance by impairing the ability to apply this strategy. However, if the strategy must be applied in a familiar context hand placement may have no effect, or it could even enhance performance (given the numerical advantage seen in Experiment 2).

The notion that hand placement can have differential effects on mathematics performance serves as an important reminder of the complex relationship between cognitive functioning, on the one hand, and performance on higher-level cognitive tasks, on the other hand. What is cognitively advantageous for one task may be disadvantageous for another. As one example, high working memory capacity has been shown to benefit analytic problem solving but impair creative problem solving (Wiley & Jarosz, 2012). As another example, dyslexia has been associated with better detection of visual anomalies in complex perceptual scenes (Schneps, Brockmole, Sonnert, & Pomplun, 2012; Schneps, Rose, & Fischer, 2007). While the "double-edged sword" of cognition may not be lost on cognitive psychologists, it is frequently overlooked in the culture of education. As body-based approaches to STEM education begin to gain traction with students, teachers, and policymakers, it will be critical to ensure that their application is not "one size fits all." Our vision for the future of action-based approaches to learning and performance is based on the idea of finding the right match between one's actions/postures and the demands of the learning context. With this in mind, learners may better achieve desired outcomes by teaching them to be mindful and strategic about how and when they engage their bodies.

Notes

i) While it is certainly true that not all techniques that fall under the umbrella of "hands-on learning" involve the overt use of the hands, many do (see Kontra, Lyons, Fischer, & Beilock, 2015, for a recent example involving learning of physics concepts). By the same token, there are many ways in which students use their hands during learning that would not be considered "hands-on learning" in the conventional sense (e.g., a student who reads from a hard copy instead of a desktop computer). The key point we wish to make here is that *hand placement* is a critical factor for any learning scenario in which a meaningful spatial relationship can be established between the hands and learning materials, regardless of whether that scenario involves "hands-on learning" as it is conventionally defined in pedagogy.

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Competing Interests

The authors have declared that no competing interests exist.

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Data Availability

The data for both experiments is freely available (see the Supplementary Materials section).

Supplementary Materials

The Supplemental Materials include the data for both experiments (for access see Index of Supplementary Materials below).

Index of Supplementary Materials

Davoli, C. C., O’Rear, C. D., McAulay, E., McNeil, N. M., & Brockmole, J. R. (2020). *Supplementary materials to "Hand position affects performance on multiplication tasks"* [Research data]. PsychOpen. https://doi.org/10.23668/psycharchives.3018

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