

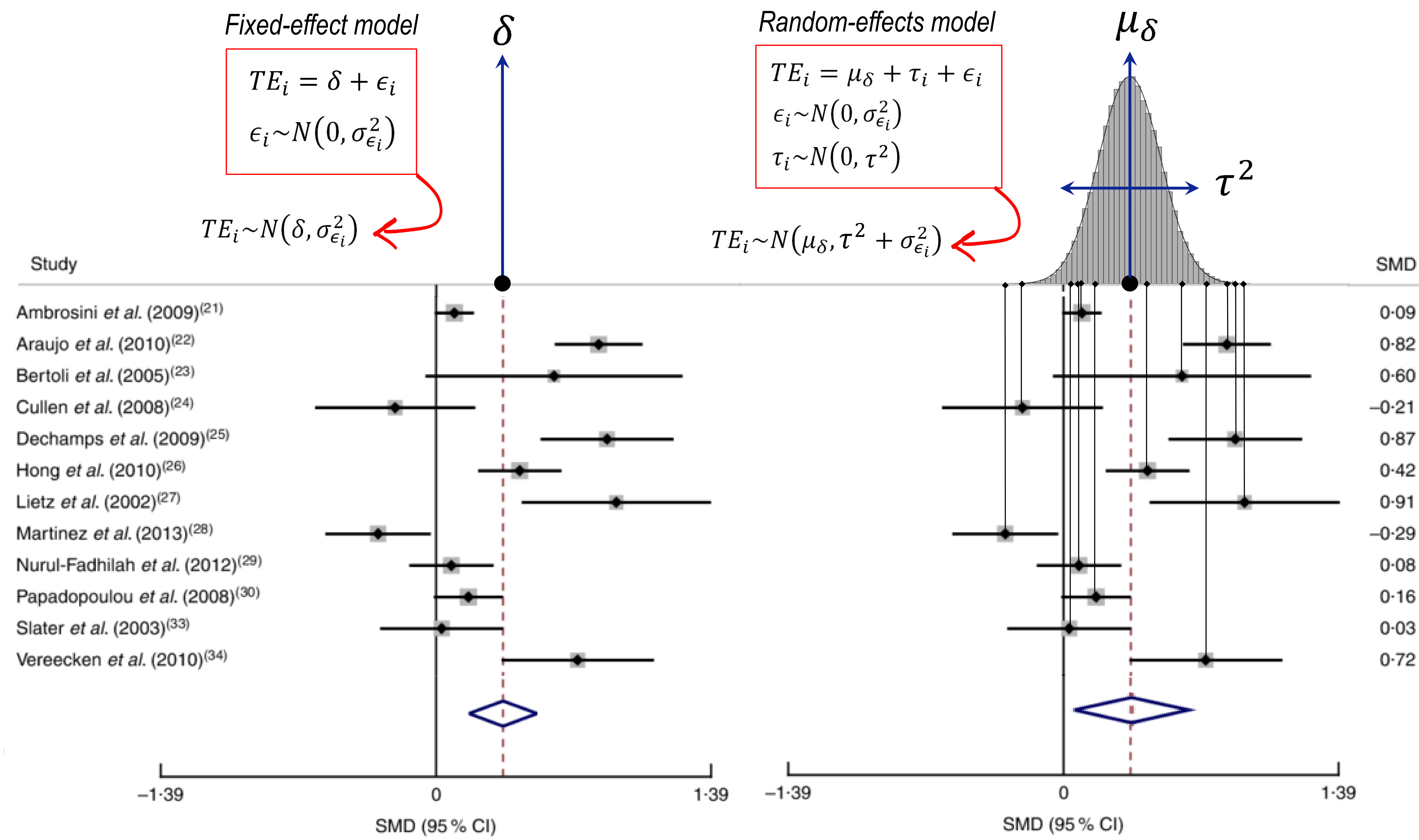
ROBUSTNESS OF DIFFERENT ESTIMATORS OF THE BETWEEN-STUDY VARIANCE IN RANDOM EFFECTS META-ANALYSES: A MONTE CARLO SIMULATION

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Forest plot obtained from Tabacchi, G., Filippi, A. R., Amodio, E., Jemni, M., Bianco, A., Firenze, A., & Mammina, C. (2016). A meta-analysis of the validity of FFQ targeted to adolescents. *Public Health Nutrition*, 19(7), 1168-1183.

Point estimator for τ^2		Author (year)	Computation	Image	Estimation method
Cochran (also known as Hedges-Olkin)	CA	Cochran (1954)	Direct	Non-negative	Method of the moments
Paule-Mandel	PM	Paule & Mandel (1982)	Iterative	Non-negative	
DerSimonian-Laird	DL	DerSimonian & Laird (1986)	Direct	Non-negative	
Hartung-Makambi	HM	Hartung & Makambi (2000)	Direct	Positive	
Two-step Cochran	CA2	DerSimonian & Kacker (2007)	Direct	Non-negative	
Two-step DerSimonian-Laird	DL2	DerSimonian & Kacker (2007)	Direct	Non-negative	
Improved Paule-Mandel	IPM	Bhaumik et al. (2012)	Iterative	Non-negative	
Positive DerSimonian-Laird	DLp	Kontopantelis et al. (2013)	Direct	Non-negative	
Lin-Chu-Hodges r	LCHr	Lin, Chu and Hodges (2017)	Direct	Non-negative	
Lin-Chu-Hodges m	LCHm	Lin, Chu and Hodges (2017)	Direct	Non-negative	
Multistep DerSimonian-Laird	DLm	vanAert & Jackson (2018)	Direct	Non-negative	
Approximated restricted maximum likelihood	ARML	Morris (1983)	Iterative	Non-negative	Maximum likelihood
Maximum likelihood	ML	Hardy & Thompson (1996)	Iterative	Non-negative	
Restricted maximum likelihood	RML	Viechtbauer (2005)	Iterative	Non-negative	
Sidik-Jonkman	SJ	Sidik & Jonkman (2005)	Direct	Positive	Least squares
Sidik-Jonkman (prior CA estimation)	SJca	Sidik & Jonkman (2007)	Direct	Positive	
Empirical bayes (equivalent to PM estimation)	EB	Morris (1983)	Iterative	Non-negative	Bayesian
Fully bayesian	FB	Smith, Spiegelhalter & Thomas (1995)	Iterative	Positive	
Rukhin	RB	Rukhin (2013)	Iterative	Non-negative	
Rukhin positive	RBp	Rukhin (2013)	Iterative	Positive	
Bayes Modal	BM	Chung et al. (2013)	Iterative	Positive	
Non-parametric bootstrap DerSimonian & Laird	DLbs	Kontopantelis et al. (2013)	Direct	Non-negative	Non-parametric
Malzahn, Böhning y Holling	MBH	Malzahn, Böhning & Holling (2000)	Direct	Non-negative	
Hunter-Schmidt (unweighted, equivalent to CA)	HSuw	Hunter & Schmidt (2004)	Direct	Non-negative	Artifact correction
Hunter-Schmidt (weighted by sample size)	HSs	Hunter & Schmidt (2004)	Direct	Non-negative	
Hunter-Schmidt (weighted by inversed variance)	HSiv	Hunter & Schmidt (2004)	Direct	Non-negative	

<i>Interval estimator for τ^2</i>	<i>Point estimators associated</i>		<i>Author (year)</i>	<i>Computation</i>
Bayesian highest posterior density interval	Fully bayesian	BCI	Box & Tiao (1973)	Iterative
Parametric bootstrap	All	PB	Efron & Tibshirani (1993)	Direct
Non-parametric bootstrap	All	NPB	Efron & Tibshirani (1993)	Direct
Profile likelihood	Maximum likelihood	PML	Hardy & Thompson (1996)	Iterative
Wald-type	Maximum likelihood	WML	Biggerstaff & Tweedie (1997)	Direct
Biggerstaff-Tweedie	None	BT	Biggerstaff & Tweedie (1997)	Iterative
Q-profile	None	QP	Hartung & Knapp (2005)	Iterative
Sidik-Jonkman	SJ	SJ	Sidik & Jonkman (2005)	Direct
Modified Q-profile	None	MQP	Knapp, Biggerstaff & Hartung (2006)	Iterative
Sidik-Jonkman (prior CA estimation)	SJca	SJca	Sidik & Jonkman (2007)	Direct
Profile likelihood	Restricted maximum likelihood	PRML	Vietchbauer (2007)	Iterative
Wald-type	Restricted maximum likelihood	WRML	Vietchbauer (2007)	Direct
Generalized variable approach	None	GV	Tian (2008)	Iterative
Biggerstaff & Jackson	None	BJ	Biggerstaff & Jackson (2008)	Iterative
Jackson	None	J	Jackson (2013)	Iterative
Approximate Jackson	Method of moments estimators	AJ	Jackson, Bowden & Baker (2015)	Direct
Unequal-tail Q-profile	None	UTQP	Jackson & Bowden (2016)	Iterative

[...]

Veroniki, A.A., Jackson, D., Viechtbauer, W., Bender, R., Bowden, J., Knapp, G., Kuss, O., Higgins, J.P.T., Langan, D., & Salanti, G. (2015). Methods to estimate the between-study variance and its uncertainty in meta-analysis. *Research Synthesis Methods*, 7(1), 55–79.

Langan, D., Higgins, J. P. T., & Simmonds, M. (2016). Comparative performance of heterogeneity variance estimators in meta-analysis: A review of simulation studies. *Research Synthesis Methods*, 8, 181-198.

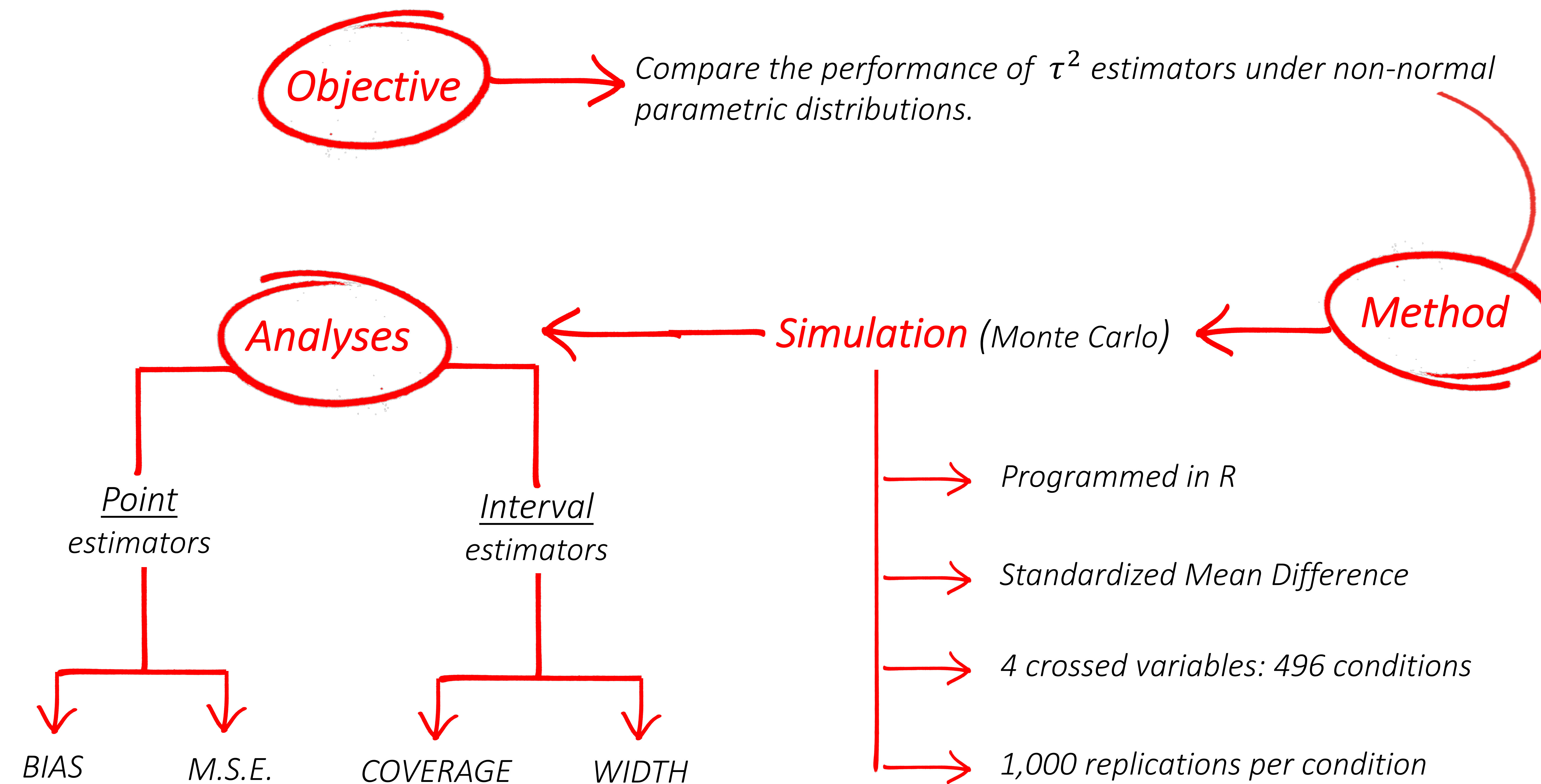
Petropoulou, M., & Mavridis, D. (2017). A comparison of 20 heterogeneity variance estimators in statistical synthesis of results from studies: a simulation study. *Statistics in Medicine*, 36(27), 4266-4280.

Langan, D., Higgins, J.P., Jackson, D., Bowden, J., Veroniki, A.A., Kontopantelis, E., Viechtbauer, W., & Simmonds, M. (2019). A comparison of heterogeneity variance estimators in simulated random-effects meta-analyses. *Research Synthesis Methods*, 10(1):83–98.

van Aert, R.C., van Assen, M.A., & Viechtbauer, W. (2019). Statistical properties of methods based on the Q-statistic for constructing a confidence interval for the between-study variance in meta- analysis. *Research Synthesis Methods*, 10(2), 225–239.

Veroniki, A. A., Jackson, D., Bender, R., Kuss, O., Langan, D., Higgins, J. P., ... & Salanti, G. (2019). Methods to calculate uncertainty in the estimated overall effect size from a random-effects meta-analysis. *Research Synthesis Methods*, 10(1), 23-43.

Zhang, C., Chen, M., & Wang, X. (2020). Statistical methods for quantifying between-study heterogeneity in meta-analysis with focus on rare binary events. *Statistics and Its Interface*, 13(4), 449-464.



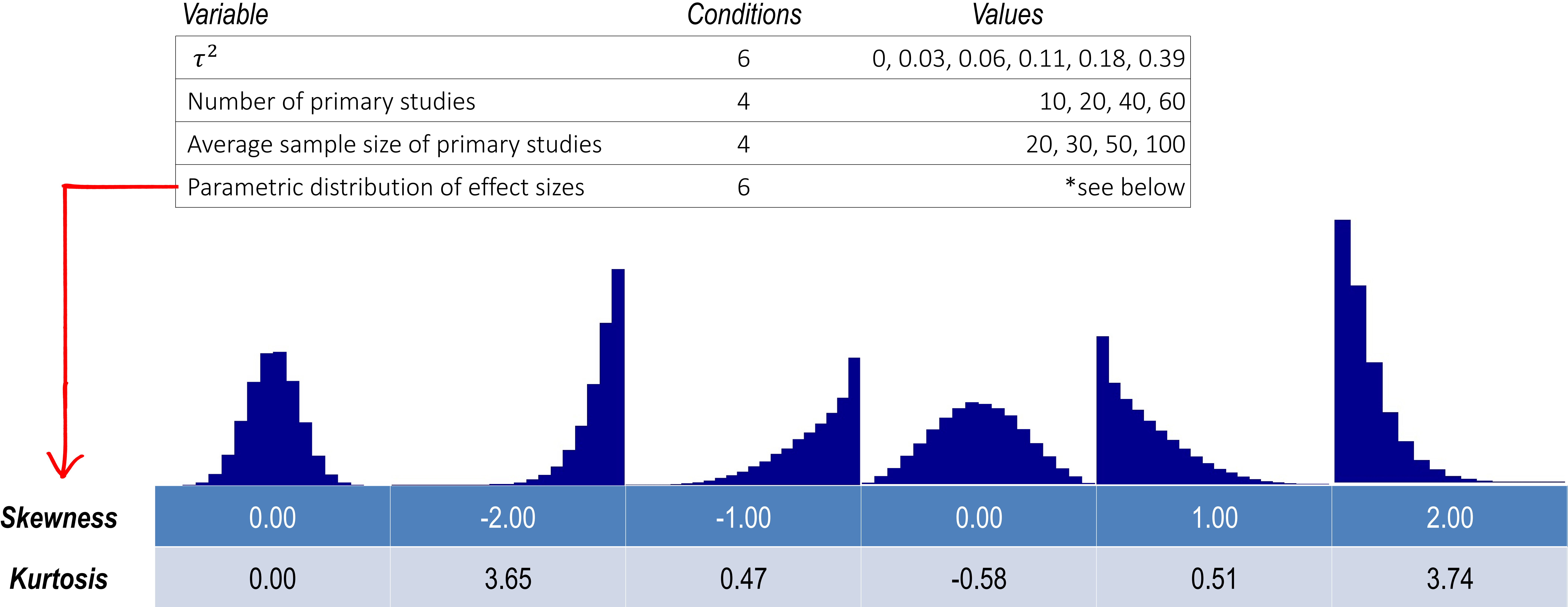


Figure 1. Pair of skewness and kurtosis values for each condition of the simulation variable *Parametric distribution of effect sizes*.

Empirical basis

Rubio-Aparicio et al. (2018). A methodological review of meta-analyses of the effectiveness of clinical psychology treatments. *Behavior Research Methods*, 50(5), 2057-2073.

Skewness
& kurtosis
condition

LESS

BIAS

MORE

1°

2°

3°

4°

5°

6°

7°

8°

9°

10°

11°

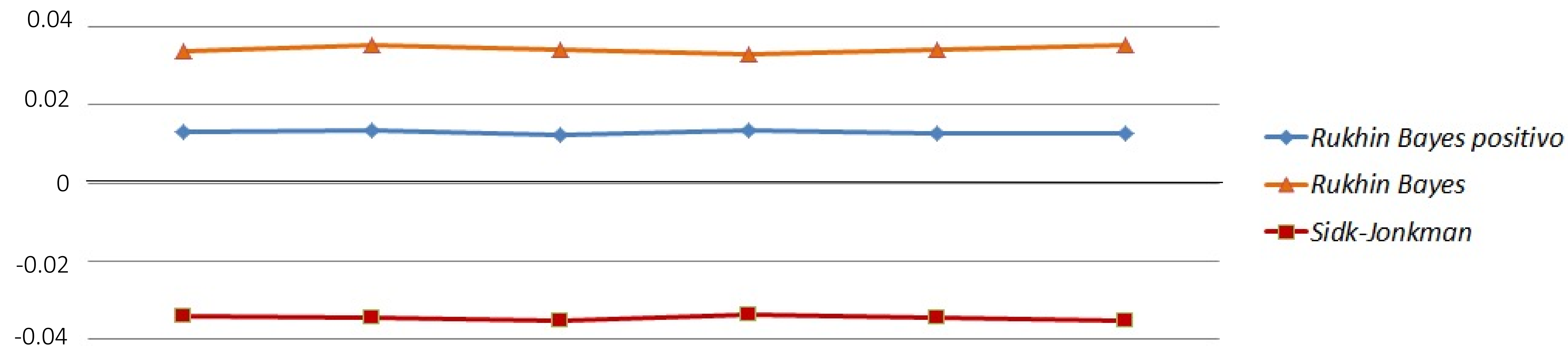
12°

13°

14°

1	RBp .0130	RBz .0338	SJ -.0342	HO -.0627	HM -.0649	HO2 -.0661	PM -.0663	DL2 -.0665	DLp -.0681	REML -.0698	DL -.0725	ML -.0767	HS -.0791	MBH .1680
2	RBp .0135	SJ -.0343	RBz .0354	HO -.0599	HO2 -.0662	PM -.0667	DL2 -.0676	HM -.0707	REML -.0733	DLp -.0742	DL -.0788	ML -.0803	HS -.0850	MBH .1737
3	RBp .0124	RBz .0341	SJ -.0350	HO -.0630	HM -.0664	HO2 -.0669	PM -.0671	DL2 -.0674	DLp -.0697	REML -.0711	DL -.0742	ML -.0780	HS -.0806	MBH .1684
4	RBp .0134	RBz .0330	SJ -.0335	HM -.0625	HO -.0628	HO2 -.0654	PM -.0655	DLp -.0656	DL2 -.0656	REML -.0680	DL -.0699	ML -.0751	HS -.0767	MBH .1657
5	RBp .0127	RBz .0344	SJ -.0345	HO -.0624	HM -.0660	HO2 -.0663	PM -.0665	DL2 -.0668	DLp -.0692	REML -.0705	DL -.0737	ML -.0774	HS -.0802	MBH .1681
6	RBp .0128	SJ -.0351	RBz .0352	HO -.0605	HO2 -.0670	PM -.0675	DL2 -.0684	HM -.0716	REML -.0743	DLp -.0751	DL -.0798	ML -.0811	HS -.0859	MBH .1747

Table 1. Point estimators for τ^2 ordered according to their bias as a function of the skewness and kurtosis exhibited by the parametric distribution of effect sizes.



Skewness	0.00	-2.00	-1.00	0.00	1.00	2.00
Kurtosis	0.00	3.65	0.47	-0.58	0.51	3.74

Figure 2. Bias of the Rukhin Bayes, positive Rukhin Bayes, and Sidik-Jonkman estimates as a function of the skewness and kurtosis exhibited by the parametric distribution of effect sizes.

Skewness & kurtosis condition	<div><div>LESS</div><div>M.S.E.</div><div>MORE</div></div>													
	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°
1	RBp	SJ	HM	DLp	HO	HO2	DL2	PM	DL	REML	HS	ML	RBz	MBH
	.0056	.0062	.0089	.0102	.0103	.0106	.0107	.0107	.0107	.0111	.0115	.0117	.0682	.2633
2	HM	RBp	SJ	DLp	DL	HS	DL2	ML	REML	PM	HO2	HO	RBz	MBH
	.0120	.0121	.0126	.0137	.0144	.0149	.0149	.0168	.0170	.0171	.0172	.0178	.0566	.3217
3	RBp	SJ	HM	DLp	HO	DL	DL2	HO2	PM	REML	HS	ML	RBz	MBH
	.0065	.0071	.0095	.0109	.0113	.0114	.0116	.0116	.0116	.0120	.0122	.0126	.0866	.2693
4	RBp	SJ	HM	HO	DLp	HO2	DL2	PM	DL	REML	HS	ML	RBz	MBH
	.0043	.0048	.0077	.0088	.0089	.0091	.0091	.0091	.0093	.0095	.0102	.0103	.0512	.2470
5	RBp	SJ	HM	DLp	DL	HO	DL2	HO2	PM	REML	HS	ML	RBz	MBH
	.0065	.0071	.0094	.0108	.0113	.0114	.0115	.0116	.0116	.0120	.0121	.0125	.0832	.2701
6	HM	RBp	SJ	DLp	DL	HS	DL2	ML	REML	PM	HO2	HO	RBz	MBH
	.0121	.0123	.0127	.0138	.0144	.0150	.0167	.0169	.0171	.0172	.0173	.0179	.0571	.3331

Table 2. Point estimators for τ^2 ordered according to their mean squared error as a function of the skewness and kurtosis exhibited by the parametric distribution of effect sizes.

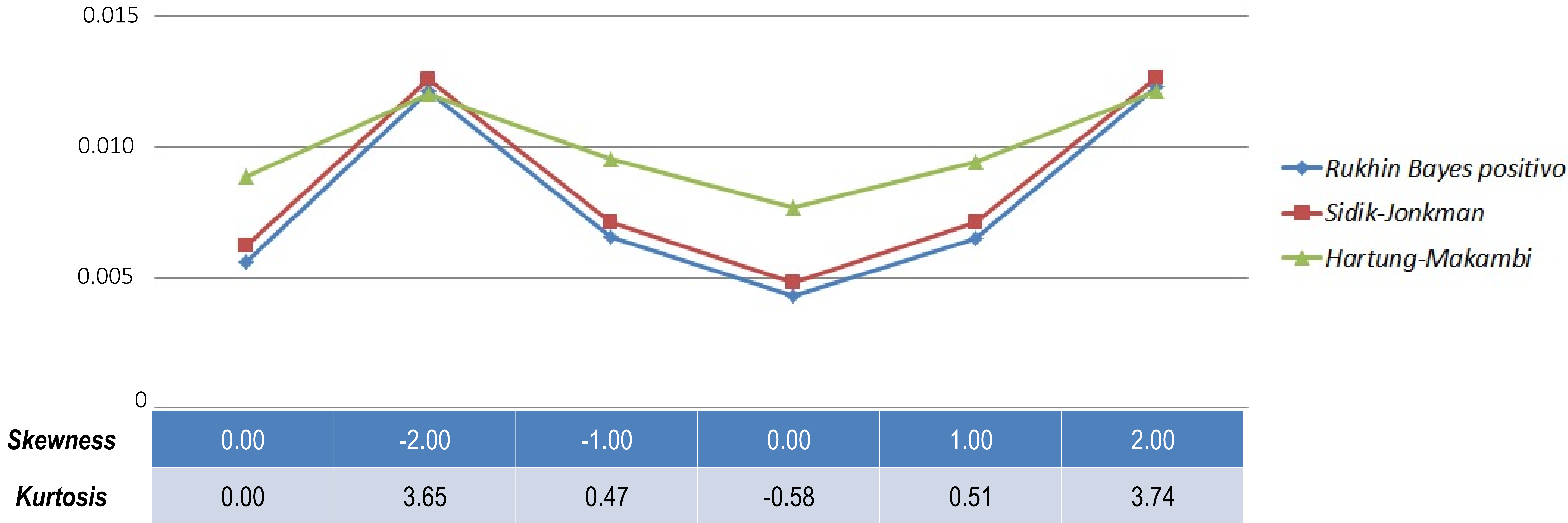


Figure 3. Mean squared error of the positive Rukhin Bayes, Sidik-Jonkman, and Hartung and Makambi estimates as a function of the skewness and kurtosis exhibited by the parametric distribution of effect sizes.

Skewness and kurtosis condition		BT	BJ	J	QP	WT(RBp)	WT(SJ)	SJ(SJ)
Coverage (%)	1	54.92	54.9	56.15	56.31	98.34	62.03	62.4
	2	47.61	47.59	49.41	49.36	94.14	54.58	52.5
	3	53.37	53.35	54.77	54.87	97.37	60.65	60.73
	4	57.56	57.54	58.49	58.53	99.04	65.07	65.45
	5	52.98	52.96	54.34	54.44	97.47	60.13	60.11
	6	47.05	47.04	48.77	48.8	93.98	53.8	51.77
Width	1	.2386	.2288	.2368	.2437	.2958	.1647	.1844
	2	.2280	.2162	.2306	.2448	.2966	.1657	.1865
	3	.2385	.2245	.2334	.2412	.2946	.1634	.1825
	4	.2440	.2351	.2412	.2463	.2970	.1658	.1861
	5	.2348	.2265	.2355	.2435	.2956	.1646	.1846
	6	.2246	.2129	.2272	.2415	.2948	.1640	.1839

Tables 3 and 4. Coverage percentage of the actual τ^2 value and interval confidence width of the Biggerstaff-Tweedie (BT), Biggerstaff-Jackson, (BJ), Jackson (J), Q-profile (QP), Wald-type (WT), and Sidik-Jonkman (SJ) estimates as a function of the skewness and kurtosis exhibited by the parametric distribution of effect sizes.

In summary...

What have we done?

- We have simulated databases following real meta-analyses.
- We have manipulated the parametric distribution of effect sizes
- We have compared 14 point estimators and 7 confidence intervals for τ^2 .

What have we got?

Point estimators:

- Non-normal scenarios do not affect the bias, but there is a trend in efficiency.
- The most unbiased and efficient point estimator has been shown to be positive Rukhin Bayes.

Interval estimators:

- The only interval estimator associated with positive Rukhin Bayes is Wald-type, and its coverage is close to 95%.
- Non-normal scenarios do affect the coverage probability, but not the interval width.

But...

What are the limitations of our work?

- Those inherent to simulations.
- Lack of point estimators.
- Lack of interval estimators.
- Credible intervals are not considered.
- Results only generalizable to meta-analyses with SMD or other asymptotically normal effect size indices.

Thanks!

Any questions?

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