

Supplementary information: Developing a Rigorous Measure of the Pre-school Home Mathematics Environment

Appendix 1. Pre-school Home Mathematics Questionnaire (PHMQ)

Instructions: Please complete the following questionnaire, answering all questions. This questionnaire will take approximately 15 minutes to complete. **These questions are in relation to your child who is aged 3-4 years.** **Please tick or circle** the choice that best describes **your family**.

ABOUT YOU

1. What age are you? _____

2. What is your relationship to the participating child?

(a) Mother	
(b) Stepmother	
(c) Father	
(d) Stepfather	
(e) Grandparent	
(f) Foster parent	
(g) Adoptive parent	
(h) Other, please state:	

3. What is your current marital status?

(a) Single (never married)	
(b) Married	
(c) Cohabiting (not married)	
(d) Divorced	
(e) Separated	
(f) Widowed	

4. Are you the primary carer? (e.g. Spend most of the time with the child)

(a) Yes	
(b) No	

5. What is your ethnic origin?

(a) Asian	
(b) Black or African American	
(c) White, Caucasian	
(d) Chinese	
(e) Mixed	
(f) Other, please state:	

6. What is the first language you speak with your child?

(a) English	
(b) Irish	
(c) Spanish	
(d) French	
(e) Polish	
(f) Other, please state:	

7. What is your highest educational qualification?

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other, please state:	

8. What is your highest level of mathematical achievement? (Including any degree that involves statistical training)

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other, please state:	

9. Are you currently employed?

If **currently** employed proceed to question 12.

(a) Yes full-time	
(b) Yes part-time	
(c) No	

10. If no, have you previously been employed? If **previously** employed proceed to question 12.

(a) Yes	
(b) No	

11. If no, do you provide full-time child-care?

If **full-time carer**, please proceed to question 20.

(a) Yes	
(b) No	

Details of current/previous employment

12. What is/was your main job title?

13. What activities do/did you mainly do in your job?

14. What does/did the firm/organisation you worked for mainly make or do? (e.g. Provide leisure services, retail industry, education)

15. Are/were you working as an employee or are/were you self-employed?

(a) Employee	
(b) Self-employed	

If Employee – Go to question 16
If Self-employed – Go to question 18

Employee only

16. In your job, do/did you have any formal responsibility for supervising the work of other employees?

(a) Yes	
(b) No	

17. How many people work/worked for the employer at the place where you work/worked?

(a) 1 to 10	
(b) 11 to 24	
(c) 25 to 499	
(d) 500 or more employees	

Please continue to question 20

Self-employed only

18. Are/were you working on your own or do/did you have employees?

(a) On own	
(b) With partner	
(c) No employees	
(d) Employees	
(e) Other:	

19. If you have/had employees, how many people do/did you employ at the place where you work/worked?

(a) 1 to 10	
(b) 11 to 24	
(c) 25 to 499	
(d) 500 or more employees	

OTHER ADULTS LIVING IN HOUSEHOLD

20. Are there other adults living in your household?

(a) Yes	
(b) No	

If Yes – please continue
If No – Go to question 23

21. Person' s relationship to child?

(a) Mother	
(b) Stepmother	
(c) Father	
(d) Stepfather	
(e) Grandparent	
(f) Foster parent	
(g) Adoptive parent	
(h) Other, please state:	

22. What is this adults highest educational qualification?

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other, please state:	

ABOUT YOUR PARTICIPATING CHILD

These questions are in relation to your child who is aged 3-4 years.

23. When was your child born? ___ / ___ / ____ (Day/Month/Year)

24. Including the child in question, how many children do you have in total?

Total number of children: _____

25. What is the birth order of your participating child aged 3 - 4?

(a) Only child	
(b) First born (oldest)	
(c) Second born	
(d) Third born	
(e) Fourth born	
(f) Fifth born	
(g) Other, please state:	

26. What is your participating child' s gender?

(a) Male	
(b) Female	

27. How many languages can your participating child speak?

(a) One	
(b) Two	
(c) Other, please state:	

28. What are these languages? _____

29. Are you interested in maths as a topic?

(a) No, I am completely disinterested in maths	
(b) No, I am disinterested in maths	
(c) Neither interested or disinterested in maths	
(d) Yes, I am interested in maths	
(e) Yes, I am very interested in maths	

30. Ideally, how much education would you want your participating child to complete?

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other, please state:	

31. Ideally, what would you want your participating child's highest mathematical achievement to be?

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other, please state:	

MATHS LITERACY

32. In the past month, how often did you and your child engage in reading? ***Please circle***

activity did not occur few times a month about once a week few times a week almost daily

| _____ | _____ | _____ | _____ | _____ |

33. Do any of the books you read to the participating child involve numbers?

(a) Yes	
(b) No	

If Yes - How many?
_____ (give as number)

34. Would you do maths activities more or less than reading?

(a) More	
(b) Less	
(c) Same	

NUMERACY

35. How high can your child currently count up to?

36. Did you ask your child to count to answer the above question?

(a) Yes	
(b) No	

37. How high do you think a child at your child's age should be able to count?

38. Who is more likely to bring up numeracy activities?

(a) You	
(b) Your child	
(c) Both	
(d) Other:	

39. Imagine you have asked your child a sum and they get the answer wrong, what are the specific things you say or do to encourage and support your child to learn maths?

<i>Please order the following options</i> in the order you would use each. <i>1 - 'most likely' to 4 - 'least likely'</i> <i>Please do not leave any blank</i>	<i>Insert number below</i>	<i>Example:</i>
(a) Question and encourage your child without explanation (e.g. "No that's not the right answer, what number do you think it would be?")		1
(b) Prompt, explain and work through the problem together (e.g. Make sure he/she understand where they went wrong)		2
(c) Provide answer and move on		3
(d) Adjust your behaviour (e.g. demonstrate visually with objects/fingers)		4

FREQUENCY OF HOUSEHOLD ACTIVITIES

40. In the past month, how often did you and your child engage in the following? *Please circle*

1. Counting

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

2. Write numbers

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

3. Scenarios number games (e.g. “If I have two toy cars and I take one away, how many cars do I have?”)

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

4. Counting on fingers/hands

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

5. Watching number related TV shows (e.g. Number Jacks or Numtums)

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

6. Teaching about measurements (e.g. baking, height)

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

7. Sticker books

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

8. Sorting shapes

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

9. Rhyming TV shows involving numbers (e.g. Number Jacks)

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

10. Play with jigsaws

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

11. Watch educational programs (e.g. Dora the Explorer)

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

12. Sorting objects by size

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

13. Comparing sets of objects (e.g. brother has more than mum)

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

14. Pairing/matching games

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

15. Playing with building blocks

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

16. Identifying names of written numbers

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

17. Counting out food, dinner plates, knives and forks

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

18. Creating patterns with objects (e.g. arranging blocks into shapes)

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

19. Counting objects (e.g. ducks in bath, blocks, new toys, books)

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

20. Teaching about money (e.g. informal - playing shop or formal - buying sweeties)

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

21. Time terminology (e.g. big hand, little hand)

activity did not occur few times a month about once a week few times a week almost daily
|-----|-----|-----|-----|

22. Asking shape related questions (e.g. “how many sides does a circle have?”)

activity did not occur	few times a month	about once a week	few times a week	almost daily
_____	_____	_____	_____	_____

TECHNOLOGY

41. The following questions are all relating to technology usage (computers, tablets, smart phones). If your child **does not** use technology, please go to question 42.

In the past month, how often did your child engage in the following? ***Please circle***

1. Maths applications (e.g. Number Jacks)

activity did not occur	few times a month	about once a week	few times a week	almost daily
_____	_____	_____	_____	_____

2. Maths related websites (e.g. coolmaths.com)

activity did not occur	few times a month	about once a week	few times a week	almost daily
_____	_____	_____	_____	_____

3. Racing games (e.g. the faster they complete sums, the faster the boat moves)

activity did not occur	few times a month	about once a week	few times a week	almost daily
_____	_____	_____	_____	_____

4. Size/matching apps (e.g. “put the big skirt on the small girl”)

activity did not occur	few times a month	about once a week	few times a week	almost daily
_____	_____	_____	_____	_____

5. Add and subtraction games

activity did not occur	few times a month	about once a week	few times a week	almost daily
_____	_____	_____	_____	_____

6. Filling in the gap number games (e.g. what is next in the sequence?)

activity did not occur	few times a month	about once a week	few times a week	almost daily
_____	_____	_____	_____	_____

7. Maths related YouTube videos (e.g. NumTums)

activity did not occur	few times a month	about once a week	few times a week	almost daily
_____	_____	_____	_____	_____

BOARD GAMES

42. Below you will see a list of games for nursery children. Some of these are popular children’s games, and some are made up.

Please read the names and put a tick next to those games that you know to be real games.

Do not guess, but only tick those you know.

It is extremely important that you answer without stopping to verify any games.

(a) Battleships	
(b) Beach Shelter	
(c) Buckaroo	
(d) Build A Beetle	
(e) Chasin' Cheeky	
(f) Croc Doctor	
(g) Crocodile Dentist	
(h) Doctor Pop-up	
(i) Dog Tales	
(j) Doh Nutters Game	
(k) Dominoes	
(l) Elefun	
(m) Exasperation	
(n) Frustration	
(o) Guess who?	
(p) Head to toe	
(q) Hungry Hungry Hippo	
(r) Kerplunk	
(s) Ludo	
(t) Mailman	
(u) Mashup	
(v) Monopoly Junior	
(w) Operation	
(x) Pepper Pigs	
(y) Pie Face	
(z) Pop-up Pirate	
(aa) Shark Chase	
(bb) Snakes and Ladders	
(cc) Spider Web Master	
(dd) The Mashin Max Game	

SIBLINGS

43. Do you feel that your child has learnt number skills from their siblings?

(a) Yes	
(b) No	
(c) Does <u>not</u> apply	

44. When your children are doing activities together that involve maths, what types of activities are they most likely to do together? Keeping this in mind, in the past month, how often have you and your child engage in the following? ***Please circle***

1. Counting objects together

activity did not occur few times a month about once a week few times a week almost daily
 |-----|-----|-----|-----|

2. Arranging objects by size, shape or colour

activity did not occur few times a month about once a week few times a week almost daily
 |-----|-----|-----|-----|

3. Watching number related TV shows together (e.g. Number Jacks or Numtums)

activity did not occur few times a month about once a week few times a week almost daily
 |-----|-----|-----|-----|

4. Sing rhyming songs together (e.g. "1, 2, 3, 4, 5 once I caught a fish alive")

activity did not occur few times a month about once a week few times a week almost daily
 |-----|-----|-----|-----|

5. Reading books together that involve numbers (e.g. Hungry Caterpillar)

activity did not occur few times a month about once a week few times a week almost daily
 |-----|-----|-----|-----|

6. Timed games (e.g. hide and seek)

activity did not occur few times a month about once a week few times a week almost daily
 |-----|-----|-----|-----|

7. Everyday activities that involve number (e.g. using money while shopping)

activity did not occur few times a month about once a week few times a week almost daily
 |-----|-----|-----|-----|

Thank you for taking the time to fill in this questionnaire!

Appendix 2. Summary of Items from PHMQ

Table 1. Summary of Items, how they were Generated and Initial Item Reduction Criteria (before final PHMQ was created)

Qu No. from the original PHMQ	Items with home numeracy dimension category	Stage 1: Generation; Inductive Deductive approach *	Item or overlap literature	Stage 1: items with **	Stage 3: Initial Item Reduction; Kept or Removed	Stage 3: Initial Item Reduction; Reason for removal
Parent expectations – Benchmark questions						
30	Ideally, how much education would you want your participating child to complete?	Inductive			Kept	/
31	Ideally, what would you want your participating child's highest mathematical achievement to be?	Inductive			Kept	/
Literacy – Benchmark questions						
32	In the past month, how often did you and your child engage in reading?	Deductive	LeFevre et al., 2009		Kept	/
33	Do any of the books you read to the participating child involve numbers?	Inductive			Kept	/
33a	If Yes – How many?	Inductive			Kept	/
34	Would you do maths activities more or less than reading?	Inductive			Kept	/
Numeracy – Benchmark questions						
35	How high can your child currently count up to?	Deductive	LeFevre et al., 2009		Kept	/
36	Did you ask your child to count to answer the above question?	Deductive	LeFevre et al., 2009		Kept	/
37	How high do you think a child at your child's age should be able to count?	Inductive			Kept	/
Parent-child interaction – Interaction questions						
38	Who is more likely to bring up numeracy activities?	Inductive			Kept	/
39	What are the specific things you say or do to encourage and support your child to learn maths?				Kept	/
39a	Question and encourage your child without explanation	Deductive	Vandermaas-Peeler et al., 2012		Kept	/
39b	Prompt, explain and work through the problem together	Deductive	Vandermaas-Peeler et al., 2012		Kept	/

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39c	Provide answer and move on	Deductive	Vandermaas-Peeler et al., 2012	Kept	/
39d	Adjust your behaviour	Deductive	Vandermaas-Peeler et al., 2012	Kept	/
Frequency of household activities					
40	In the past month, how often did you and your child engage in the following?				
1	Counting	Deductive	Melhuish et al., 2008	Kept	/
2	Feeding objects (e.g. posting letters)	Inductive		Removed	EFC
3	Hopscotch	Inductive		Removed	EFC
4	Write numbers	Deductive	LeFevre et al., 2009	Kept	/
5	Scenarios number games (e.g. "If I have two toy cars and I take one away, how many cars I have?")	Deductive	LeFevre et al., 2009; Lukie, Skwarchuk, LeFevre & Sowinski., 2014	Kept	/
6	Counting on fingers/hands	Inductive		Kept	/
7	Watching number related TV shows (e.g. Number Jacks or Numtums)	Inductive		Kept	/
8	Teaching about measurements (e.g. baking, height)	Deductive	LeFevre et al., 2009; Lukie, Skwarchuk, LeFevre & Sowinski., 2014	Kept	/
9	Sticker books	Inductive		Kept	/
10	Counting out turn taking (e.g. jumping to ten on trampoline)	Inductive		Removed	EFC
11	Sorting shapes	Deductive	LeFevre et al., 2009; Kleemans, Peeters, Segers & Verhoevena., 2012; Lukie, Skwarchuk, LeFevre & Sowinski., 2014	Kept	/

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12	Rhyming TV shows involving numbers (e.g. Number Jacks)	Inductive		Kept	/
13	Using number cards (e.g. order the cards by number)	Deductive	LeFevre et al., 2009; Lukie, Skwarchuk, LeFevre & Sowinski., 2014	Removed	EFC
14	Play with jigsaws	Inductive		Kept	/
15	Rhyming storybooks (e.g. Dr Seuss)	Inductive		Removed	EFC
16	Dot-to-dot number books	Deductive	LeFevre et al., 2009	Removed	EFC
17	Watch educational programs (e.g. Dora the Explorer)	Deductive	LeFevre et al., 2009	Kept	/
18	Sorting objects by size	Deductive	LeFevre et al., 2009	Kept	/
19	Counting up stairs	Inductive		Removed	EFC
20	Comparing sets of objects (e.g. brother has more than mum)	Inductive		Kept	/
21	Pairing/matching games	Inductive		Kept	/
22	Play card games (e.g. "jack change it")	Deductive	LeFevre et al., 2009	Removed	EFC
23	Playing with building blocks	Deductive	LeFevre et al., 2009	Kept	/
24	Identifying names of written numbers	Deductive	LeFevre et al., 2009	Kept	/
25	Counting out food, dinner plates, knives and forks	Inductive		Kept	/
26	Rhyming songs including counting (e.g. "1, 2, 3, 4, 5 once I caught a fish alive" or "ten green bottles")	Deductive	Kleemans, Peeters, Segers & Verhoevena., 2012; Melhuish et al., 2008	Removed	EFC
27	Creating patterns with objects (e.g. arranging blocks into shapes)	Inductive		Kept	/
28	Being timed (e.g. hide and seek)	Deductive	LeFevre et al., 2009	Removed	EFC
29	Counting objects (e.g. ducks in bath, blocks, new toys, books)	Deductive	LeFevre et al., 2009	Kept	/

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30	Teaching about money (e.g. informal – playing shop or formal – buying sweeties)	Deductive	LeFevre et al., 2009	Kept	/
31	Time terminology (e.g. big hand, little hand)	Deductive	Lukie, Skwarchuk, LeFevre & Sowinski., 2014	Kept	/
32	Asking shape related questions (e.g. “how many sides does a circle have?”)	Inductive		Kept	/
Frequency of technology					
41	In the past month, how often did you and your child engage in the following?				
1	Maths applications (e.g. Number Jacks)	Inductive		Kept	/
2	Maths related websites (e.g. coolmaths.com)	Inductive		Kept	/
3	Racing games (e.g. faster they complete sums the faster the boat moves)	Inductive		Kept	/
4	Size/matching apps (e.g. “put the big skirt on the small girl”)	Inductive		Kept	/
5	Add and subtraction games	Inductive		Kept	/
6	Filling in the gap number games (e.g. what is next in the sequence?)	Inductive		Kept	/
Siblings – Interaction questions					
42	Do you feel that your child has learnt number skills from their siblings?	Deductive	Benigno et al. (2004)	Kept	/
43	What would your participating child (aged 3 – 4) be more likely to do when engaged in a mathematical based activity with siblings?	Inductive		Removed	Lack of variation in responses
44	When your children are interacting mathematically, what types of activities are they most likely to do together?				
44a	Counting objects together	Inductive		Kept	/
44b	Arranging objects by size, shape or colour	Inductive		Kept	/
44c	Observing older siblings homework	Inductive		Removed	Lack of variance; Least likely to occur in the home
44d	Taking part in older siblings homework	Inductive		Removed	Lack of variance; Least likely to occur in the home
44e	Maths applications on technology device (e.g. Playing Number Jacks on iPhone)	Inductive		Removed	Lack of variance; Least likely to occur in the home

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44f	Watching number related TV shows together (e.g. Number Jacks or Numtums)	Inductive	Kept	/
44g	Sing rhyming songs together (e.g. "1, 2, 3, 4, 5 once I caught a fish alive")	Inductive	Kept	/
44h	Reading books together that involve numbers (e.g. Hungry Caterpillar)	Inductive	Kept	/
44i	Play board games or card games together (e.g. "jack change it")	Inductive	Removed	Lack of variance; Least likely to occur in the home
44j	Timed games (e.g. hide and seek)	Inductive	Kept	/
44k	Everyday activities that involve number (e.g. using money while shopping)	Inductive	Kept	/
Understanding				
45	Do you believe that your child understands the meaning of number words up to 5?	Inductive	Removed	Lack of variation in responses
46	Do you believe that your child understands the meaning of odd and even?	Inductive	Removed	Lack of variation in responses
47	Do you believe that your child understands the meaning of more and less?	Inductive	Removed	Lack of variation in responses
Support				
48	Do you believe it is important for caregivers to support numeracy learning in the home?	Inductive	Removed	Lack of variation in responses

Note: * Inductive items = 44 items; Deductive items = 25; Total items = 69. ** Inductive items removed = 14; Deductive items removed = 5; Total items after removal = 50.

Appendix 3. Content Validity

Table 2. Content Validity: Dimensions (or Subscales) from PHMQ and Sample Commentary from Interviews

Dimension (or subscales) with definitions	Examples of items within the dimensions from PHMQ	Examples from interviews
Structure of the home numeracy environment		
<p>Parent expectation of their children's academic success and child counting ability dimensions</p> <p>This theme is different in that it does not reflect one specific dimension of the PHMQ, moreover, it reflects two PHMQ dimensions and the balance between structured/formal and unstructured/informal numeracy environments. This was the same theme that was found in the original interviews (Cahoon et al., 2017)</p>	<p>How high can your child currently count up to?</p> <p>How high do you think a child at your child's age should be able to count?</p> <p>Who is more likely to bring up numeracy activities?</p>	<p>"He seems to be excelling at maths. He loves the counting and will do it himself now and he is only 3. I can hear him when he is on his own counting out figures, counting out Peppa Pig and separating things... even his Shreddies and Cheerio this morning for breakfast he counted those. So, we can nearly be counting all day without realising you're doing it, with nearly everything" – Participant 1</p> <p>"When we are walking places it's easier to count things like how many red cars are there, so when you are out numeracy would be easier" – Participant 2</p> <p>"I haven't gone out of my way to get numeracy games it's just everyday objects" – Participant 5</p>
Child maths literacy		
<p>Child maths literacy</p> <p>The evidence suggests that children are accessing number learning through books.</p>	<p>In the past month, how often did you and your child engage in reading?</p> <p>Do any of the books you read to the participating child involve numbers?</p> <p>If so, how many?</p> <p>Would you do maths activities more or less than reading?</p>	<p>"If you're reading a book and if there is a picture, she'll say "Look mummy there's three dogs" or she'll count them "One, two, three" from the picture. There probably is more numeracy than literacy at the minute just because she is quite young" – Participant 2</p> <p>"Sometimes we would find things in books, sometimes I will say "find..." he is really into pirate so "find five swords in the picture". In that instance I suppose there is that element of counting when he is searching for things, that's quite frequent actually" – Participant 4</p> <p>"She'll count, she has a pirate book. When you get it right you push the button and it makes a little noise. The first page is 'Count which arrow has four diamonds' and there's one arrow with 3 dots and an arrow with 4 diamonds. She counts the one with the dots 1, 2, 3 and then she</p>

		will count the one with the 4 diamonds. They are very close together and she can still go 1, 2, 3, 4 and then she will press the button when she gets it right” – Participant 7
Parent-child teaching methods		
Parent-child teaching methods	Who is more likely to bring up numeracy activities?	“Lately it has been, “I’ve got 3 fingers on this hand and I’ve got 4 fingers on this hand, together that equals?”. Very showy stuff rather than in your head” – Participant 5
Each rank order option was mentioned in the current interviews.	What are the specific interactions the parent/guardian would do to encourage and support the target child to learn numeracy? with 4 rank order options (e.g. question and encourage your child without explanation, prompt, explain and work through the problem together, provide answer and move on and adjust your behaviour).	“Ella has these four dollies and I would say “Now Ella you have four dollies you could give two of those you Rob”. It’s working with items and visualising numbers, but practically as well” – Participant 6 “She loves counting, she’s really good at counting, she would count up to 20 and then I would try to do “One and add another one, what does that make?” (moved objects to demonstrate) but she’s not really getting it yet, she is too little” – Participant 7
Target child-sibling interactions		
Target child-sibling interactions	When your children are doing activities together that involve maths, what types of activities are they most likely to do together? Keeping this in mind, in the past month, how often have you and your child engage in the following?	“Her older brother is interested in maths. I would say she is maybe following his lead. She has an IKEA kitchen in the living room and I hear him counting sometimes. Then when he is at school I can hear her counting things just because that’s what he does” – Participant 2
There were a wide variety of activities occurring between parent, target child, and older sibling/s, (i.e. triad interactions).	Do you feel that your child has learnt number skills from their siblings?	“Most of the numeracy between the two of them would be about sharing. How much Rob has compared to how much she has and how to make it the same” – Participant 6 “Amy is his older half-sister. They interact well considering the age gap. Amy would be very good, she would be a lot better than me, at going through things like colour. I would say she has taught Jake colours and she would go through the days of the week with him too” – Participant 8
Frequency of maths activities scale		
1. Parent –child interaction Any number-based interaction between the primary parent/guardian and their child in	1. Write numbers 2. Scenarios number games (e.g. “If I have two toy cars and I take one away, how many cars I have?”)	“They’ll (target child and older sibling) play together with Play Doh but there is usually a bit of a dispute if you leave them together alone. It’s better if adults play with him than any of his peer group. He is still at

<p>the home. Activities were a parent is necessary for the child to learn from the activity.</p>	<ol style="list-style-type: none"> 3. Teaching about measurements (e.g. baking, height) 4. Sticker books 5. Identifying names of written numbers 6. Teaching about money (e.g. informal – playing shop or formal – buying sweets) 7. Time terminology (e.g. big hand, little hand) 8. Asking shape related questions (e.g. “how many sides does a circle have?”) 	<p>the solidity play, well a bit of parallel play, but he’s not moved onto co-operating” – Participant 1</p> <p>“If we are baking I would try and get her to count the bun cases” – Participant 2</p> <p>“She loves jigsaws. It’s always supervised with mummy, and me going “You find another piece of Ariel’s tail for me” but she loves it” – Participant 6</p>
<p>2. Computer maths games Any computer - based activities (such as, tablet or smartphone usage) that occur in the home, specifically games that involve number, shape or problem solving.</p>	<ol style="list-style-type: none"> 1. Maths applications (e.g. Number Jacks) 2. Maths related websites (e.g. coolmaths.com) 3. Racing games (e.g. faster they complete sums the faster the boat moves) 4. Size/matching apps (e.g. “put the big skirt on the small girl”) 5. Add and subtraction games 6. Filling in the gap number games (e.g. what is next in the sequence?) 	<p>“On the iPad, he does the shadow into the shape, the racing games, and the one with the balloons on the number train” – Participant 1</p> <p>“There’s a Cbeebies app and the games on that are all educational” – Participant 2</p> <p>“This EduKitchen app is good. So, there’s a recycling bin and they pick up all the rubbish. They would have fruit and then wrappers to work out which ones go in the recycling bin so it is quite educational” – Participant 4</p>
<p>3. TV programmes Any educational TV programmes watched in the home involving rhymes and/or numbers.</p>	<ol style="list-style-type: none"> 1. Watching number related TV shows (e.g. Number Jacks or Numtums) 2. Rhyming TV shows involving numbers (e.g. Number Jacks) 3. Watch educational programmes (e.g. Dora the Explorer) 	<p>“TV can be a great motivator. You can say to them if we finish this then we’ll put on Peppa Pig. It’s great because they’ll complete it before they go and watch TV” – Participant 1</p> <p>“He prefers cartoons but he does watch things like Mr Tumble and Gigglebizz. and there is Kerwhizz too. It’s a game show with aliens and ask number, shape or what’s missing questions” – Participant 4</p> <p>“I’d rather them watch the Numtums (than non-educational TV), I think it’s quite good” – Participant 5</p>
<p>4. Shape Any shape, pattern or sorting based activity in the home.</p>	<ol style="list-style-type: none"> 1. Sorting shapes 2. Play with jigsaws 3. Sorting objects by size 4. Pairing/matching games 5. Playing with building blocks 6. Creating patterns with objects (e.g. arranging blocks into shapes) 	<p>He does the game with the wooden shapes, where you fit them into the holes and he loves matching cards like animal dominos where you match all the cows together” – Participant 4</p> <p>“She’s good at jigsaws. She knows to do the straight edge, she’ll work from the corner. She has an 8 piece, 12 piece, 18 piece and a 24-piece jigsaw. She can do the 24 piece, she might need help. The smaller</p>

		ones she can do on her own but the larger ones she'll need a bit of help to get started" – Participant 7
5. Counting Activities that involve the counting or comparing of objects in the home.	1. Counting 2. Counting on fingers/hands 3. Comparing sets of objects (e.g. brother has more than mum) 4. Counting out food, dinner plates, knives and forks 5. Counting objects (e.g. ducks in bath, blocks, new toys, books)	"We would do puzzles together, jigsaws, and you can see his progression with more pieces now" - Participant 8 "She will count on her own without me prompting her. She's very particular, almost an OCD level where everything has to be exact, she's very exact when she comes to counting" – Participant 3 "He looks forward to his bedtime stories. In fairness, he gets to pick stories and now and again we'd say well you've been good so pick out 4 and he would go out and pick out 4 books. He picks out 2 books normally" – Participant 5 "She sits and count away to herself whenever she is playing, but she can only reliably count to 10 and then it becomes 33 and 54 and random numbers" – Participant 6

Appendix 4. Criterion Validity

Table 3. Criterion Validity: Subscale Dimensions and Sample Commentary from Interviews

Subscale dimension with definitions	Frequency items	Examples from interviews
1. Parent –child interaction	Low parent-child interaction	<p>“He would help me bake now and again, not too often because of all the mess that comes with it but now and again he would help me cook and measure out ingredients” – Participant 5 (M = .63)</p> <p>“I suppose our main focus would be colours rather than numbers” – Participant 5 (M = .63)</p> <p>“It’s not something that I have thought about (asking number-related questions while reading) but he got a homework book back, and there was a question in it about “what age do you think the girl is?” and he had to count the balloons. It wouldn’t be something that I would have thought of” – Participant 5 (M = .63)</p>
	High parent-child interaction	<p>“We play with Play-Doh, rolling it up in balls, squashing it and counting it. This brings up counting and the shapes” – Participant 1 (M = 2.75)</p> <p>“We’ll count the animals” (in a book they own at home) – Participant 1 (M = 2.75)</p> <p>“Obviously in the evening she’s a bit tired and it’s more fun rather than learning, and in the early afternoon when she’s finished nursery we’ll try and re-enforce what she has learnt that day whether it be the alphabet or numbers; any kind of homework” – Participant 3 (M = 2.38)</p>
2. Computer maths games	Low computer maths games	<p>“In this day and age there is more portable media and I worry how that would affect her learning. I think overuse of the game will affect her imagination, that creativity, that’s why we limit it to maybe an hour a day at the very most” – Participant 3 (M = 1.00)</p> <p>“I prefer the games to be educational... The Edukitchen app is really good and the Cbeebies app is good too, because it makes him think. Furchester hotel as well. There is a problem he has to solve in each room and there are three ways he can solve the problem” – Participant 4 (M = .83)</p>

	High computer maths games	<p>“There is a Cbeebies app that I downloaded and it’s for learning. He does colour in and counting activities on it” – Participant 8 (M = 2.00)</p> <p>“I would probably get half an hour’s peace out of the Cbeebies app, whereas when he watches Batman on YouTube I would get an hour” – Participant 8 (M = 2.00)</p>
3. TV programmes	Low TV programmes	Programmes with non – learning outcomes: “She likes a bit of My Little Pony but mostly Paw Patrol. Oh and Disney films, she loves Rapunzel and she loves Frozen” – Participant 6 (M = 1.00)
	High TV programmes	<p>Programmes with learning outcomes: “She loves PJ masks, Peppa Pig, Lazy town, Numtums and Octonauts” – Participant 7 (M = 3.00)</p> <p>Programmes with learning outcomes: “The Cbeebies TV shows do have numbers because there’s Numtums and Squiggle It too... I’d rather them watch Numtums and stuff like that. I think it’s good for learning” – Participant 5 (M = 3.00)</p>
4. Shape	Low shape	<p>“I don’t do that many structured activities. If they wanted to do painting or building blocks or do a jigsaw I would sit with them” – Participant 6 (M = 1.33)</p> <p>“He used to play jigsaws quite often before Rachel was born. He used to be very focused he would have sat and done a jigsaw and I actually thought he was quite smart at one point because he was doing the bigger jigsaws, bigger wooden ones that have 48 pieces” – Participant 5 (M = 1.67)</p>
	High shape	<p>“She loves Jenga. Jenga’s her new favourite game. Sometimes we build houses with the Jenga block but she does quite like playing Jenga, pushing the blocks out. She’s actually quite good at it” – Participant 2 (M = 3.00)</p> <p>“She loves puzzles. She loves jigsaws. She has lots of jigsaws” – Participant 2 (M = 3.00)</p>
		“He’s great at matching, we match beads and bags of pegs. Also in my sewing box we’ll sort buttons into big, medium, small piles” – Participant 1 (M = 4.00)

		<p>“To keep him engaged if you change the visual object, he thinks it’s something new... he’ll identify and sort out by colour and then he’ll count” – Participant 1 (M = 4.00)</p>
5. Counting	Low counting	<p>“We would count the stairs and he would be counting along with me, but we’ve always been 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. It’s just rote learning at the moment not sums” – Participant 5 (M = 1.60)</p> <p>“We count every day whether it be steps or how many things are in front of her because I do want her to start learning but it’s probably a bit early to do like subtraction with her or anything like adding” – Participant 7 (M = 2.40)</p>
	High counting	<p>“She likes counting. If we are out somewhere, she will count flowers, or she will count dogs, like “They have two dogs.”” – Participant 2 (M = 3.60)</p> <p>“She would count things spontaneously, be it if she’s jumping up and down or the number of cows in a field, we live near a field, she’ll count the cows’ without me telling her too” – Participant 3 (M = 3.60)</p> <p>“She doesn’t get as much time on her own as he did (older son) but she picks up a lot of things from him, so a lot of her spontaneous counting out objects is because he does it and she is copying him” – Participant 2 (M = 3.60)</p>

Appendix 5. Confirmatory Factor Analysis for the Frequency of Maths Activities

Model one: One factor, total frequency of maths activities

Model one theorises a single factor construct consisting of all items measuring the frequency of maths activities. A one-factor model is also known as a g-factor or general factor model (Geiser, 2012). This model is based upon previous works that have measured the home numeracy environment through a unidimensional construct (Blevins-Knabe et al., 1996; Kleemans et al., 2012) where all activities occurring in the home environment related to maths development have been measured. This unidimensional approach provides a general overview of the influence of the home numeracy environment however, it does not give specific information or understanding of what types of activities and environments enhance early numeracy skills. The rationale for this model was to compare it against the five-factor model found through the exploratory factor analysis in the previous chapter to confirm which model was the superior fit.

Model three: Five-factor second order

Model three (Figure 1) is based on the concept that the five-factor model can be incorporated into two single factors; (1) *interaction with the parent*, constituted by the parent-child interaction, shape and counting subscales, and (2) *no interactions with the parent*, which includes computer maths games and TV programmes subscales. The three-factors, parent-child interaction, shape and counting were strongly correlated with each other ($r = .77$ to $.88$, Table 25, Figure 6). This indicates that there was little discriminant validity among the three-factors. Despite the strong correlations among the three-factors, the fit statistics indicated that more than one factor was needed to sufficiently account for the observed variances and covariance's of the five frequencies of numeracy-activities subscales and that a g-factor solution (Model one, Wang & Wang, 2012) had to be rejected for the current study. Even though the three-factors appear to share a substantial amount of common variance, there also is a nontrivial portion of systematic variance that needs to be taken into account and that explains why the correlations among the three-factors differ from 1.0 (Geiser, 2012).

Table 4. Pearson Zero-Order Correlations Between Subscales on the Frequency of Numeracy-Activities Scale (Model two: Five-Factor Model)

	1. Counting	2	3	4
2. Parent-child interaction	.881**			
3. TV programmes	.313**	.288*		
4. Shape	.785**	.773**	.381**	
5. Computer maths games	.283**	.481**	.437**	.411**

Note: * $p < .05$ ** $p < .01$ (two-tailed).

Thus, a five-factor second order model was theorised. This model hypothesises the use of a second order model, where the second order factors account for the variation among the first order factors (Wang & Wang, 2012). This model allows for the dimensionality of the frequency of numeracy-related activities to be explored further beyond the use of first order factors. In this model, three-factors in the five-factor model load onto a factor and the other two-factors in the five-factor model load onto another factor, so-called second order factors. One factor was given the name *interaction with the parent* as the activities in the parent-child interaction, shape and counting subscales could all involve interaction with the parent. The other two-factors, computer maths games and TV programmes, moderately correlated with each other ($r = .44$) and although this is not a strong enough correlation to assume that they are one second order factor it was deemed that these activities did not necessarily involve interactions with parents, as children are interacting with electronic devices. Thus, the two-factors, computer maths games and TV programmes, were named *no interaction with the parent* in the second order factor. Therefore, the five-factors are dichotomised into numeracy-related

activities that could involve interactions with parents (parent-child interaction, shape and counting) and those activities that may not involve interactions with parents (computer maths games and TV programmes) in the second order model.

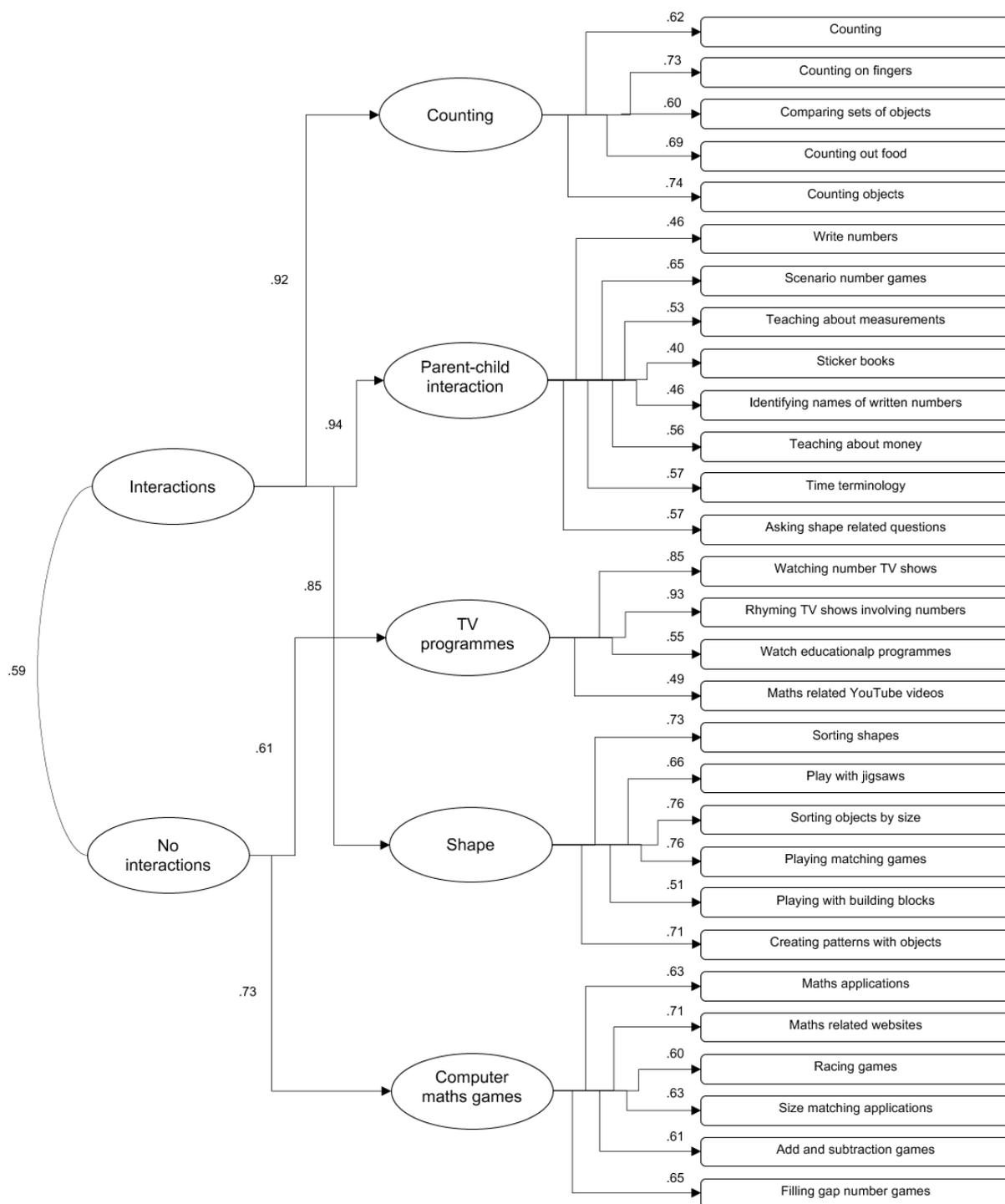


Figure 1. Five-Factor Second Order Model of the Frequency of Number Activities Scale in the PHMQ

Model four: Two-factor model based on the original definitions of direct and indirect numeracy activities by LeFevre et al. (2009)

Model four (Figure 2) is based on LeFevre et al. (2009) definitions of direct and indirect numeracy activities. The definitions set by LeFevre et al. (2009) suggested that *direct* activities, involving explicit and intentional teaching about numbers, quantity, or arithmetic to develop children's numeracy skills (e.g. counting objects) and *indirect* activities, involve numbers in real-world tasks (e.g. playing board games with dice) that include 'hidden' mathematical instructions that occur incidentally. LeFevre et al. (2009) found a four-factor structure, using a confirmatory factor analysis, which was labelled; (1) number skills (e.g. counting objects), (2) number books (e.g. "connect-the-dot" activities), (3) games (e.g. playing card games) and (4) applications (e.g. measuring ingredients when cooking). LeFevre et al. (2009) classified that those activities reported in the number skills and number books subscales reflected *direct* teaching activities and the games and application factors reflected *indirect* experiences. Thus, in the current study the researcher arbitrarily assigned the items into two subscales, direct and indirect numeracy activities to help with the selection of the best fitting model. The items assignment procedure for this model is included in Appendix 6.1 Items assignment procedure for Model 4, the items in each category can be seen in Figure 8. The rationale for this model was to compare it against the five-factor model hypothesised in the previous Chapter to help with the selection of the best fitting model.



Figure 2. Two-Factor Model Based on the Original Definitions by LeFevre et al. (2009)

Best fit model.

The selection of the most appropriate model was based upon goodness of fit statistics (see Table 3 in main manuscript). Models one and four failed to reach acceptable model fit with low CFI values .57 and TLI values .53. However, models two and three were the most acceptable model fit indices with each reporting a CFI of .81 and a TLI of .79. Good fitting models are indicated by a CFI of $> .95$ (better model: $> .97$) and the same cut-off value for TLI applies (Geiser, 2012). A CFI $> .90$ is often regarded as an indicator of an adequate model fit (Hair et al., 2010; Coroiu et al., 2018; Awang, 2012) the same cut-off value for TLI applies (Forza & Filippini, 1998; Coroiu et al., 2018; Awang, 2012).

The CFI and the TLI are incremental fit indices that compare the fit of the target model to the fit of a baseline model (Geiser, 2012). In Mplus the baseline model, also known as the null independence model, assumes that the population covariance matrix of the observed variables is a diagonal matrix, in other words it is assumed that there is no relationship between any of the variables (Geiser, 2012). As a consequence, it is possible that the null model is "too good", meaning that the average level of correlations in the current data is rather low. In this case, Kenny (2012) argued that CFI should not be computed if the RMSEA of the null model is less than 0.158 as the CFI obtained will be too small a value (Kenny & McCoach, 2003; Beldhuis, 2012). When investigating the RMSEA values both models two and three demonstrate acceptable RMSEA values (< 0.08) (Awang, 2012), however model two has a lower RMSEA of .073 compared to model three that had a RMSEA of .074. Therefore, a slightly better RMSEA value was demonstrated for model two.

The SRMR coefficient is a standardised measure for the evaluation of the model residuals, however SRMR is somewhat biased by sample size. Marsh, Hau and Wen (2004) state that the SRMR values for solutions based on small sample sizes are unacceptable (greater than .08), whereas those based on large sample sizes are acceptable. A value $< .08$ is generally considered a good fit (Hu & Bentler, 1999). Therefore, given that model two presents the most acceptable fit indices (CFI = .81, TLI = .79, RMSEA = 0.073, SRMR = 0.078), it seems reasonable that a five-factor model, which includes parent-child interaction, computer maths games, TV programmes, shape and counting, to be deemed the most suitable measurement model. In order to obtain the best model, fit a closer examination of the factor loadings between models two and three was necessary. Slightly higher standardised factor loadings were demonstrated for model two than model three. With regards to the information criteria, BIC, AIC and adjusted BIC, model three has a smaller BIC (11753.563), but model two has a

smaller AIC (11480.253) and ABIC (11455.925). Nevertheless, model two is the preferred model when taking into consideration all fit criteria for assessing goodness of fit.

Table 5. Model Fit Statistics for the Alternative Models of Frequency of Maths Activities (n = 136)

Model no.	Model explained	$\chi^2(p)$	df	CFI	TLI	RMSEA (90% CI) p	SRMR	AIC	BIC	Sample-Size Adjusted BIC
1	One factor model	992.714 (0.00)	377	0.57	0.53	0.110 (0.101 – 0.118) 0.00	0.109	11841.537	12094.938	11819.717
2	Five-factor model	633.871 (0.00)	367	0.81	0.79	0.073 (0.063 – 0.083) 0.00	0.078	11480.253	11762.780	11455.925
3	Five-factor second order model	644.616 (0.00)	371	0.81	0.79	0.074 (0.064 – 0.083) 0.00	0.081	11482.686	11753.563	11459.361
4	Two-factor model (original direct and indirect activities)	992.703 (0.00)	376	0.57	0.53	0.110 (0.102 – 0.118) 0.00	0.109	11842.808	12099.121	11820.737

Note: Estimator = MLR; n = 136; χ^2 = Chi-square Goodness of Fit statistic; *df* = degrees of freedom; p = Statistical significance; CFI = Comparative Fit Index; TLI = Tucker Lewis Index; RMSEA (90% CI) = Root-Mean-Square Error of Approximation with 90% confidence intervals; BIC = Bayesian Information Criterion; AIC = Akaike information criterion