

Supplementary Materials

Section A – Simulation Code Used in the Study

```
install.packages(distr)
install.packages(truncdist)
install.packages(boot)
install.packages(MASS)
install.packages(psych)
options(scipen=999)
set.seed(33883838)
labn <- 'C:/'
library(distr)
library(truncdist)
library(boot)
library(MASS)
library(psych)
rtoa <- function(r){
  return(asin(r)/pi+0.5)}
ator <- function(a){return(sin(pi*(a-0.5)))}
archtanh <- function(r){0.5*log((1+r)/(1-r))}
tanh <- function(z){(exp(2*z)-1)/(exp(2*z)+1)}
pb<- function(data, conf.level = .95, bootno = 2000, seed.no = 38383388){
  rtoa <- function(r){
    return(asin(r)/pi+0.5)}
  n <-length(data[,1])
  pb.est<-function(data){
    n <-length(data[,1])
    meanx <- mean(data[,1])
    meany <- mean(data[,2])
    diff <- (data[,1]-meanx)*(data[,2]-meanx)
    tcount <- length(which(diff>0))
    PBS <- tcount/n
    return(PBS)}
  PBS <- pb.est(data)
  q <- 2*PBS-1
  SE<- sqrt(0.25*((1-q^2)/n))
  ll <- qnorm((1-conf.level)/2)*SE+PBS # symmetrical confidence interval when lambda^2, i.e., 1.96^2 << n
  ul <- qnorm(1-(1-conf.level)/2)*SE+PBS
  p <- 2*pnorm(-abs((PBS-0.5)/SE))
  tll <- qt((1-conf.level)/2,(n-2))*SE+PBS
  tul <- qt(1-(1-conf.level)/2,(n-2))*SE+PBS
  tp <- 2*pt(-abs((PBS-0.5)/SE),df=n)
  ##bootstrap##
  funbp <- function(d, i){
    d2 <- d[i,]
    return(pb.est(d2))}
  bootr <- boot(data, funbp, R = bootno)
  bci <- boot.ci(bootr, conf = conf.level, type = c("perc", "bca"))
  sebsi <- sd(bootr[[2]])
  bsill <- qnorm((1-conf.level)/2)*sebsi+bci[[2]]
  bsiul <- qnorm((1-conf.level)/2+conf.level)*sebsi+bci[[2]]
  bsillT <- qt((1-conf.level)/2,(n-2))*sebsi+bci[[2]]
  bsiulT <- qt(1-(1-conf.level)/2,(n-2))*sebsi+bci[[2]]
  return(list(PBS = PBS, SE = SE, Lower_Limit_Z = ll, Upper_Limit_Z = ul, p_value_Z = 0,
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Lower_Limit_T = tll, Upper_Limit_T = tul , p_value_T = 0,
BSI_LL_Z = bsill, BSI_UL_Z = bsiul,
BSI_LL_T = bsillT, BSI_UL_T = bsiulT,
BPI_LL = bci[[4]][4], BPI_UL = bci[[4]][5],
BCaI_LL = bci[[5]][4], BCaI_UL = bci[[5]][5]))
}
tot <- 32
conf.level <- .95
refpt <- 0
replications <- 1000
disV <- c(1,2,3,4,5) #1: bivariate normal correlation, PBS -> 2: normal
#3: t distribution with 18 df; 4: uniform, 5: beta distribution with alpha = beta = 0.5,
pbsV <- seq(0.5,0.9,0.05)
nV <- c(20,50,100,300,500,1000)
tcond <- length(disV)*length(pbsV)*length(nV)
rep.re <- array(0, dim=c(tcond,tot))
des <- array(0, dim=c(tcond,4))
maxy <- sqrt(12)/2
miny <- -sqrt(12)/2
cond <- 1
for(dis in 1:length(disV)){
  dis <- disV[dis]
  for(pbsp in 1:length(pbsV)){
    prob <- pbsV[pbsp]
    rho <- atorn(prob)
    for(np in 1:length(nV)){
      re <- array(0, dim=c(replications,tot))
      #data matrix to save the results: PBS, SE, Coverage Prob, p, CI width, ll, ul,
      #r, SE, Coverage Prob, p, CI width, ll, ul
      n <- nV[np]
      des[cond,] <- c(dis,prob,rho,n)
      for(rep in 1:replications){
        if(dis==1){
          x<-rnorm(n,0,1)
          e<-rnorm(n,0,sqrt(1-rho^2))
          y<-rho*x+e
          data <- as.matrix(t(rbind(x,y)))}
        else{
          gen <- function(dis,refpt,n){
            y<-array(0,dim=c(n))
            de <- array(0,dim=c(n))
            de <- rbinom(n,1,prob)
            if(dis==2){x<-rnorm(n,0,1)}
            if(dis==3){
              df <- 18 # 10 observations per group
              x <- rt(n, df)}
            if(dis==4){x<-runif(n,-sqrt(12)/2,sqrt(12)/2)}
            if(dis==5){x<-rbeta(n,0.5,0.5)}
            if(refpt==0){ref=mean(x)}
            if(refpt==1){ref=median(x)}
            if(dis==2){
              abovem <- union(intersect(which(x>ref),which(de==1)),intersect(which(x<ref),which(de==0)))
              for(i in 1:n){
                if(x[i]>ref&&de[i]==1){y[i]=rtrunc(1, spec="norm", a = 0.0000000001, b = Inf)}
                if(x[i]<ref&&de[i]==1){y[i]=rtrunc(1, spec="norm", a = -Inf, b = 0)}
                if(x[i]<ref&&de[i]==0){y[i]=rtrunc(1, spec="norm", a = 0.0000000001, b = Inf)}
              }
            }
          }
          re[rep,] <- gen(dis,refpt,n)
        }
      }
    }
  }
}

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      if(x[i]>ref&&de[i]==0){y[i]=rtrunc(1, spec="norm", a = -Inf, b = 0)}
    }
  }
  if(dis==3){
    for(i in 1:n){
      if(x[i]>ref&&de[i]==1){y[i]=rtrunc(1, spec="t", a = 0.0000000001, b = Inf, df)}
      if(x[i]<ref&&de[i]==1){y[i]=rtrunc(1, spec="t", a = -Inf, b = 0, df)}
      if(x[i]<ref&&de[i]==0){y[i]=rtrunc(1, spec="t", a = 0.0000000001, b = Inf, df)}
      if(x[i]>ref&&de[i]==0){y[i]=rtrunc(1, spec="t", a = -Inf, b = 0, df)}
    }
  }
  if(dis==4){
    for(i in 1:n){
      if(x[i]>ref&&de[i]==1){y[i]=runif(1,0,maxy)}
      if(x[i]<ref&&de[i]==1){y[i]=runif(1,miny,0)}
      if(x[i]<ref&&de[i]==0){y[i]=runif(1,0,maxy)}
      if(x[i]>ref&&de[i]==0){y[i]=runif(1,miny,0)}
    }
  }
  if(dis==5){
    for(i in 1:n){
      if(x[i]>ref&&de[i]==1){y[i]=rtrunc(1, spec="beta", a = 0.5000000001, b = 1, 0.5, 0.5)}
      if(x[i]<ref&&de[i]==1){y[i]=rtrunc(1, spec="beta", a = 0, b = 0.5, 0.5,0.5)}
      if(x[i]<ref&&de[i]==0){y[i]=rtrunc(1, spec="beta", a = 0.5000000001, b = 1, 0.5,0.5)}
      if(x[i]>ref&&de[i]==0){y[i]=rtrunc(1, spec="beta", a = 0, b = 0.5, 0.5,0.5)}
    }
  }
  return(as.matrix(t(rbind(x,y))))
}
data<-gen(dis,refpt,n)
x <- data[,1]
y <- data[,2]
} # end of else
# PBS estimates
tc <- prob
checkcp <- function(ll,ul,tc){
  re<-c(0,0,0,0,0)
  if(ll < tc && ul >tc){re[1] <- 1}
  if(ll > 0.5 || ul <0.5){re[2] <- 1}
  re[3] <- ul - ll
  re[4] <- ll
  re[5] <- ul
  return(re)
}
temp <-pb(data)
re[rep,1] <- temp[[1]] #PBS estimate
re[rep,2] <- temp[[2]] #Standard error of PBS
re[rep,3:7] <- checkcp(temp[[3]],temp[[4]],tc) # coverage prob, type i error/power, width, limits based on Z
re[rep,8:12] <- checkcp(temp[[6]],temp[[7]],tc) # coverage prob, type i error/power, width, limits based on t
# Bootstrap Coverage Probabilities
re[rep,13:17] <- checkcp(temp[[9]],temp[[10]],tc) # BSI based on Z
re[rep,18:22] <- checkcp(temp[[11]],temp[[12]],tc) # BSI based on T
re[rep,23:27] <- checkcp(temp[[13]],temp[[14]],tc) # BPI
re[rep,28:32] <- checkcp(temp[[15]],temp[[16]],tc) # BCaI
}
for(i in 1:tot){
  rep.re[cond,i] <- mean(re[,i])
}
write.table(cond,paste(labn,'cond.csv',sep=""),row.names=FALSE,col.names=FALSE,sep=",")
cond <- cond+1

```

```

    }} # end of the "for" condition loops
coll <- c("PBS", "SE",
        "CP_SE_Z", "Type_1_Power_Z", "width_Z", "LL_Z", "UL_Z",
        "CP_SE_T", "Type_1_Power_T", "width_T", "LL_T", "UL_T",
        "CP_SE_BSI_Z", "Type_1_Power_BSI_Z", "width_BSI_Z", "LL_BSI_Z", "UL_BSI_Z",
        "CP_SE_BSI_T", "Type_1_Power_BSI_T", "width_BSI_T", "LL_BSI_T", "UL_BSI_T",
        "CP_SE_BPI", "Type_1_Power_BPI", "width_BPI", "LL_BPI", "UL_BPI",
        "CP_SE_BCaI", "Type_1_Power_BCaI", "width_BCaI", "LL_BCaI", "UL_BCaI"
)
colnames(re) <- coll
colnames(rep.re) <- coll
write.table(rep.re, paste(labn, 'results.csv', sep=""), row.names=FALSE, col.names=coll, sep=",")
write.table(des, paste(labn, 'design.csv', sep=""), row.names=FALSE, col.names=FALSE, sep=",")

```

Section B – Code Used in the Real-World Example

```

library(distr)
library(truncdist)
library(boot)
library(MASS)
library(psych)
rtoa <- function(r){
  return(asin(r)/pi+0.5)}
ator <- function(a){return(sin(pi*(a-0.5)))}
archtanh <- function(r){0.5*log((1+r)/(1-r))}
tanh <- function(z){(exp(2*z)-1)/(exp(2*z)+1)}
pb<- function(data, conf.level = .95, bootno = 2000, seed.no = 38383388){
  rtoa <- function(r){
    return(asin(r)/pi+0.5)}
  n <-length(data[,1])
  pb.est<-function(data){
    n <-length(data[,1])
    meanx <- mean(data[,1])
    meany <- mean(data[,2])
    diff <- (data[,1]-meanx)*(data[,2]-meanx)
    tcount <- length(which(diff>0))
    PBS <- tcount/n
    return(PBS)}
  PBS <- pb.est(data)
  q <- 2*PBS-1
  SE<- sqrt(0.25*((1-q^2)/n))
  ll <- qnorm((1-conf.level)/2)*SE+PBS # symmetrical confidence interval when lambda^2, i.e., 1.96^2 << n
  ul <- qnorm(1-(1-conf.level)/2)*SE+PBS
  p <- 2*pnorm(-abs((PBS-0.5)/SE))
  tll <- qt((1-conf.level)/2,(n-2))*SE+PBS
  tul <- qt(1-(1-conf.level)/2,(n-2))*SE+PBS
  tp <- 2*pt(-abs((PBS-0.5)/SE),df=n)
  ##bootstrap##
  funbp <- function(d, i){
    d2 <- d[i,]
    return(pb.est(d2))}
  bootr <- boot(data, funbp, R = bootno)
  bci <- boot.ci(bootr, conf = conf.level, type = c("perc","bca"))
  sebsi <- sd(bootr[[2]])
  bsill <- qnorm((1-conf.level)/2)*sebsi+bci[[2]]
  bsilul <- qnorm((1-conf.level)/2+conf.level)*sebsi+bci[[2]]
  bsillT <- qt((1-conf.level)/2,(n-2))*sebsi+bci[[2]]
  bsilulT <- qt(1-(1-conf.level)/2,(n-2))*sebsi+bci[[2]]
  return(list(PBS = PBS, SE = SE, Lower_Limit_Z = ll, Upper_Limit_Z = ul, p_value_Z = p,
    Lower_Limit_T = tll, Upper_Limit_T = tul, p_value_T = tp,
    BSI_LL_Z = bsill, BSI_UL_Z = bsilul,
    BSI_LL_T = bsillT, BSI_UL_T = bsilulT,
    BPI_LL = bci[[4]][4], BPI_UL = bci[[4]][5],
    BCal_LL = bci[[5]][4], BCal_UL = bci[[5]][5]))
}
pb(data),bootno=4000)
#data is a n by 2 matrix that saves the age and SCS scores; data can be found in Section C below.

```

#RStudio Output#

\$PBS

[1] 0.517037

\$SE

[1] 0.008601632

\$Lower_Limit_Z

[1] 0.5001781

\$Upper_Limit_Z

[1] 0.5338959

\$p_value_Z

[1] 0.04762773

\$Lower_Limit_T

[1] 0.5001721

\$Upper_Limit_T

[1] 0.533902

\$p_value_T

[1] 0.0477088

\$BSI_LL_Z

[1] 0.500247

\$BSI_UL_Z

[1] 0.5338271

\$BSI_LL_T

[1] 0.5002409

\$BSI_UL_T

[1] 0.5338331

\$BPI_LL

[1] 0.4998519

\$BPI_UL

[1] 0.5336296

\$BCaI_LL

[1] 0.5001481

\$BCaI_UL

[1] 0.5342222

Section C – Real-World Data Example

A subset of the open-access data (Raw Data from Online Personality Tests, 2018) for demonstrating the results in the section of the real-world example

age <-

c(41,50,23,42,36,29,24,35,26,43,21,39,37,64,28,46,34,31,43,47,28,21,23,22,28,23,47,61,22,36,16,40,33,16,36,30,56,22,24,49,31,51,18,46,24,36,20,18,18,45,21,23,50,26,32,18,30,32,26,45,15,30,23,27,25,39,59,41,58,22,23,26,19,27,27,26,20,16,21,64,25,51,18,43,16,39,32,19,27,20,28,30,18,40,29,24,29,33,26,40,34,37,14,38,35,19,20,18,22,35,35,28,34,41,45,22,33,30,59,23,47,34,20,28,25,22,27,18,48,37,59,36,36,26,25,42,25,23,30,44,25,27,36,18,43,19,39,32,23,28,27,25,28,40,55,48,15,29,21,29,23,15,33,18,19,19,23,20,41,19,20,33,27,41,100,23,25,24,24,22,21,22,31,37,16,28,15,25,18,18,31,23,14,56,27,23,22,33,25,23,28,32,45,31,19,30,45,40,22,46,42,23,28,21,55,15,22,34,20,24,46,24,59,25,19,43,21,56,22,48,32,20,21,65,33,33,25,47,25,33,35,26,18,32,29,27,17,15,45,55,48,38,26,30,30,22,32,24,20,25,33,30,16,27,48,56,28,25,18,35,21,18,22,29,29,77,44,19,45,34,41,26,45,28,29,29,45,29,26,18,20,16,42,40,18,20,25,32,44,34,21,20,36,29,23,22,32,19,35,49,36,44,24,16,36,29,18,24,28,42,38,15,35,57,33,15,31,21,31,61,40,48,37,21,20,40,20,15,36,40,17,21,17,27,36,42,60,39,52,24,27,23,16,24,27,34,20,18,19,49,25,18,30,19,45,24,20,38,18,24,20,36,26,22,21,25,46,25,26,30,21,28,42,30,28,24,43,16,27,18,32,24,39,26,36,19,20,20,40,42,27,28,19,31,59,22,31,33,34,26,53,21,26,27,40,38,19,19,24,24,25,24,36,44,40,35,15,25,32,19,33,21,20,26,19,14,21,40,40,28,19,25,21,24,27,60,31,38,31,15,24,44,62,56,77,35,19,18,29,18,29,27,28,37,18,22,22,48,19,21,37,25,31,34,45,26,21,24,42,31,24,18,21,22,24,27,33,39,27,17,18,25,55,27,29,19,24,33,24,22,36,23,20,41,24,24,38,19,38,46,19,26,42,18,51,71,37,37,38,20,47,24,19,52,24,45,32,20,36,78,23,42,51,49,23,23,22,20,38,20,50,34,32,23,39,30,32,20,28,27,27,18,21,24,29,20,49,23,23,32,15,34,23,25,27,22,26,25,40,31,32,46,25,33,30,17,31,30,40,24,16,61,25,27,31,29,25,34,33,33,34,21,21,26,24,21,22,21,21,22,33,26,27,25,19,47,45,46,39,17,43,18,30,24,47,22,30,32,24,44,24,25,32,22,19,34,33,22,23,16,44,48,25,24,24,21,43,20,28,35,33,44,34,54,41,31,20,25,27,41,46,17,23,21,44,39,52,46,25,40,56,28,51,16,20,23,48,54,25,54,56,42,42,25,40,27,28,22,22,42,37,17,18,57,47,37,26,48,45,32,26,29,40,18,29,36,36,20,71,32,28,63,29,52,35,31,28,22,20,26,25,31,30,48,19,31,67,20,19,31,49,30,17,24,50,48,28,20,49,21,34,32,15,19,19,25,28,39,41,34,21,31,49,25,45,51,18,44,30,22,19,35,24,30,26,23,14,20,27,26,31,28,26,17,28,22,50,27,42,25,30,32,27,32,20,26,29,22,52,42,32,32,45,26,27,34,44,68,30,17,58,15,33,41,44,57,39,25,32,21,22,72,27,20,31,33,27,47,39,52,39,26,28,29,34,55,19,21,34,26,40,30,36,31,18,24,18,72,32,38,21,29,21,22,31,33,20,35,30,28,33,24,18,25,55,46,34,18,31,40,45,21,25,25,25,22,33,28,26,42,31,48,27,33,30,21,24,55,19,23,18,21,50,27,50,38,47,34,27,35,28,24,43,21,20,30,32,18,23,27,27,18,54,28,18,32,20,31,30,39,48,28,30,14,18,37,18,21,32,28,15,33,29,32,32,18,28,47,49,25,28,25,28,24,21,22,31,19,21,52,40,26,28,19,35,44,25,41,37,28,40,21,18,38,45,21,21,42,25,42,20,32,33,22,16,27,25,26,35,22,22,25,29,46,25,50,28,24,32,24,19,26,30,35,24,47,35,44,31,23,23,43,25,27,47,20,24,38,35,22,21,22,20,34,35,20,20,19,28,19,42,36,25,33,50,19,20,39,25,38,34,30,34,43,31,55,17,26,32,23,23,30,16,27,31,27,43,19,32,19,20,29,24,19,18,23,39,27,48,23,32,25,24,21,45,38,24,35,30,28,34,17,36,62,20,20,32,28,19,25,18,21,48,41,33,32,52,27,31,28,20,21,27,28,50,43,22,40,16,21,37,40,18,19,18,21,17,28,35,50,42,51,26,27,33,24,52,19,40,59,42,35,23,34,39,28,22,64,30,23,22,33,42,41,30,31,36,27,28,28,60,17,25,25,23,19,19,18,16,23,19,38,24,18,20,50,29,46,29,26,18,18,31,25,47,42,20,22,50,27,21,46,24,45,35,24,62,20,23,16,44,21,20,28,15,23,42,47,15,31,20,23,37,47,49,17,22,42,21,42,25,43,64,24,49,32,26,22,18,18,18,54,20,21,18,17,32,23,32,19,20,20,37,29,28,32,23,52,32,26,26,50,20,20,60,18,31,43,25,29,25,18,24,63,85,36,37,20,20,30,23,24,43,27,23,25,45,16,4

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58,23,34,30,43,25,25,19,29,28,18,20,37,39,24,48,50,22,42,40,45,24,54,33,28,21,39,25,59,46,27,43,20,3
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