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Research Reports

The Relationship Between Problem Size and Fixation Patterns During Addition, Subtraction, Multiplication, and Division

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Abstract

Eye-tracking methods have only rarely been used to examine the online cognitive processing that occurs during mental arithmetic on simple arithmetic problems, that is, addition and multiplication problems with single-digit operands (e.g., operands 2 through 9; 2 + 3, 6 x 8) and the inverse subtraction and division problems (e.g., 5 – 3; 48 ÷ 6). Participants (N = 109) solved arithmetic problems from one of the four operations while their eye movements were recorded. We found three unique fixation patterns. During addition and multiplication, participants allocated half of their fixations to the operator and one-quarter to each operand, independent of problem size. The pattern was similar on small subtraction and division problems. However, on large subtraction problems, fixations were distributed approximately evenly across the three stimulus components. On large division problems, over half of the fixations occurred on the left operand, with the rest distributed between the operation sign and the right operand. We discuss the relations between these eye tracking patterns and other research on the differences in processing across arithmetic operations.

Keywords: eye tracking, mental arithmetic, problem-size effect, addition, subtraction, multiplication, division

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Basic arithmetic is important for simple computational tasks as well as a scaffold for more complex mathematical abilities. Furthermore, mental arithmetic performance is of interest to cognitive psychologists because it provides a means to examine memory (a) in an applied context, (b) in conjunction with other cognitive processes (e.g., algorithms, strategy selection), and (c) in the context of a well-defined set of stimuli. We focused on the so-called simple arithmetic facts, defined as the set of addition and multiplication problems with single-digit operands from 2 through 9 (e.g., 2 + 3; 4 x 7) and the inverse subtraction and division problems (e.g., 5 – 2; 28 ÷ 4). This ‘standard set’ of arithmetic facts has been the focus of research on simple mental arithmetic for approximately 30 years (Ashcraft & Guillaume, 2009; Campbell, 2005). People show considerable variability in arithmetic performance across operations (i.e., addition, subtraction, multiplication, and division; Campbell & Xue, 2001). Performance – as measured by response time and error rates – also varies based on the magnitude of the operands and solutions, a finding known as the *problem-size effect* (Ashcraft & Guillaume, 2009; Zbrodoff & Logan, 2005). The goal of the present research was to examine how attention is allocated to problems in different operations and of different size by measuring participants’ eye movements.

Eye-tracking methods have provided considerable insight into cognitive processes involved in visual word recognition, scene perception, and reading (see Duchowski, 2002; Rayner, 1998). Research on numerical cognition, however, has only accumulated more recently (Hartmann, 2015). Eye tracking can be useful because standard measures such as response times and error rates only provide information about the products of cognition, whereas patterns and durations of fixations reveal information about the online course of cognitive processes in real time (Hartmann, 2015). Eye-tracking measures have been used to examine a variety of simple numerical processes, including enumeration (Canfield & Smith, 1996; Li, Logan, & Zbrodoff, 2010) and counting (Hartmann, Mast, & Fischer, 2016), number representation (Schwarz & Keus, 2004), and number comparison (Huber, Mann, Nuerk, & Moeller, 2014; Merkley & Ansari, 2010; Milosavljevic, Madsen, Koch, & Rangel, 2011; Moeller, Fischer, Nuerk, & Willmes, 2009a). Eye-tracking measures have also been used to examine estimation processes, both with number lines (Schneider et al., 2008; Sullivan, Juhasz, Slattery, & Barth, 2011) and quantities (Godau, Wirth, Hansen, Haider, & Gaschler, 2014). Finally, eye-tracking measures have been used to examine more complex processes as well, including number bisection (Moeller, Fischer, Nuerk, & Willmes, 2009b), fraction processing (Huber, Moeller, & Nuerk, 2014), syntactic processing of equations (Schneider, Maruyama, Dehaene, & Sigman, 2012), and word-problem solving (de Corte, Verschaffel, & Pauwels, 1990; Hegarty, Mayer, & Green, 1992). Relatively few researchers, however, have used eye tracking to study mental arithmetic.

Suppes, Cohen, Laddaga, Anliker, and Floyd (1982, 1983; cf. Suppes, 1990) were the first to use eye-tracking measures to examine mental arithmetic. They recorded eye movements while children and adults solved complex addition and subtraction problems (e.g., 3867 + 749 + 801; 59731 – 642 – 512 – 121). Suppes et al. used the data to test predictions from a formal quantitative model of eye movements originally formulated in Suppes (1972), making an important point; an adequate model of mental arithmetic ought to predict eye movements in addition to response times and error rates.

Subsequent research using eye-tracking measures has focused on specific aspects of mental arithmetic, either using only a restricted subset of the standard set of arithmetic problems used in mathematical research or problems beyond the standard set. For example, Verschaffel, de Corte, Gielen, and Struyf (1994) recorded children's eye movements while they added three single-digit numbers (e.g., 3 + 9 + 4), finding that they fixated on the operands in the same order in which they reported adding the operands. Green, Lemaire, and Dufau (2007) replicated and extended Verschaffel et al.'s results by having adults solve three-digit, two-term addition problems (e.g., 345 + 491) while being instructed to add either the unit digits or the hundreds digits first. Participants spent more time fixating on the digits they were instructed to add first. Taken together, these two studies suggest that eye movements might be indicative of the order in which participants process the elements of arithmetic problems. However, like Suppes et al. (1982, 1983), both studies examined problems beyond the standard set.

Three studies have examined eye movements using at least a subset of the standard set of arithmetic problems. Klein, Huber, Nuerk, and Moeller (2014) measured participants' eye movements while they solved addition and subtraction problems by indicating the solution's location on a number line. Participants overestimated the solutions on addition problems and underestimated the solutions on subtraction problems. Similarly, Hartmann, Mast, and Fischer (2015) recorded participants' eye movements across a blank screen during verbal solutions to addition and subtraction problems. Eye movements moved toward the top of the screen during addition problems but not subtraction problems. The results demonstrate that spatial-numeric associations are reflected in eye movements. However, Klein et al. only measured eye movements across the number line, not the problem, and Hartmann et al. only measured eye movements in the absence of visual information. Additionally, both studies only measured eye movements during addition and subtraction.

Finally, Zhou, Zhao, Chen, and Zhou (2012) directly examined eye movements across the elements of simple arithmetic problems. They used horizontal electrooculography to examine the influence of operand order while Chinese participants performed addition and multiplication. Chinese participants frequently report mentally reversing multiplication problems in which the left operand is larger than the right operand (e.g., solving 9 x 6 as 6 x 9; LeFevre & Liu, 1997). Consistent with these reports, Chinese participants showed movements towards the larger operand first during addition but towards the smaller operand first during multiplication. The pattern of results indicates that eye-tracking measures might reflect patterns of processing that are related to problem characteristics even during relatively simple arithmetic problems. However, Zhou et al.'s methodology only allowed restricted measurement of the horizontal direction of saccades, and only examined addition and multiplication.

The existing research provides only a limited empirical record of eye movements during simple arithmetic. These gaps leave many questions open, both about the nature of eye movements during simple arithmetic and the relations between eye movements and the variables that characterize simple arithmetic performance.

One important issue that has not been addressed for simple arithmetic is whether patterns of fixations and gaze durations provide useful information about the underlying cognitive processes on these problems. Although it is typically assumed that fixation on a stimulus reflects processing of that stimulus in similar cognitive tasks such as reading (see the eye-mind assumption as outlined by Just & Carpenter, 1984; see Moeller et al., 2011 for an application to arithmetic processes), this assumption can be false depending on the nature of the cognitive task (Anderson, Bothell, & Douglass, 2004). In particular and in contrast to reading, solving simple arithmetic problems does not require participants to process the problem elements from left to right. In computer-based experiments on simple arithmetic, fixation points are typically presented in the centre of the screen at the location where the operation sign will appear, potentially forcing participants to start processing the problem with the operation sign. However, no data are available on whether participants typically start by fixating on the operation sign or with one or the other operand on simple problems. Thus, the main objective of the present research was to provide information about typical patterns of processing in relation to the common variables that influence response times and error rates.

Some evidence supports the view that participants' eye movements are influenced by the cognitive processes during arithmetic problem solving. For example, Moeller, Klein, and Nuerk (2011) examined fixation patterns in relation to problem components as participants verified multiple-digit addition problems with and without carry processes on the unit digits (e.g., 24 + 57 = 81). Problems were presented horizontally. Moeller et al. found that participants fixated longer on the unit digits as they initially processed the problem, and subsequently on the unit digit of the second operand (e.g., 7 in 57) and on the decade digit of the proposed answer (i.e., 8 in 81) than on the other elements of the problem. This pattern of increased processing on these problem elements applied to early fixations, total fixation time, and the locations of regressive saccades. One prediction in the present research that arises from this work is that large problems (which require a carry or borrow operation) might elicit increased processing of operands relative to smaller problems (which do not require carry or borrow operations).

Research on processing time of digits also provides some insight into potential sources of variability in eye movement patterns on simple arithmetic problems. Brysbaert (1995) used eye tracking to examine the processing times for numbers from 0 to 99. He showed that processing times for single numbers are influenced by number magnitude, number frequency, and number of syllables in the number word. Brysbaert also found that processing of a second number is influenced by the characteristics of a preceding number. Most notably, however, differences in number processing times for single-digit numbers were relatively small, for example, with single-digit numbers from 2 to 9 varying by approximately 5 ms per increment of difference. Brysbaert found larger effects for double-digit numbers, and for numbers that were far apart (depending on the nature of the processing task). Encoding and identification of numbers is a component of arithmetic solutions, and differences related to these processes might be most important in the early stages of processing. However, participants complete the number processing tasks used in Brysbaert's work much more quickly than arithmetic problems such as those in the present work. Accordingly, differences in encoding and identification processes might have some influence on processing times on arithmetic task, but calculation and response preparation might encompass a much larger portion of the solution latencies, as has been demonstrated in arithmetic studies where number reading times were measured separately from arithmetic solutions (e.g., Campbell, 1995; LeFevre & Liu, 1997).

In the present research, we measured eye movements while participants solved the standard set of simple arithmetic problems, that is, all combinations of operands from 2 to 9 for addition and multiplication, and the corresponding inverse problems from subtraction and division. Each participant solved problems from one of the four operations. Across operations, we compared the total gaze duration, distribution of gaze durations throughout the trial, total number of fixations, and the location of the first fixation across the two operands and the operator (e.g., 3 + 7, 12 – 9). We used total gaze duration and number of fixations to index mental calculation (see Moeller et al., 2011). The distribution of gaze durations across time intervals provided information about the patterns of processing. The location of the first fixation allowed inferences about the differences in strategic processing related to the operands because each person only solved one of the four operations. In summary, patterns for first fixations, total fixations, and gaze durations during problem solution were analyzed to explore participants' attention to problem features for each operation.

We tested several hypotheses based on the existing relevant research on eye tracking with digits and arithmetic problems. Consistent with Moeller et al. (2011), we expected processing time on the operands to increase on large problems because carry or borrow operations are often required. We also expected that participants would look longer at double- than at single-digit operands (Brysbaert, 1995). We also hypothesized that participants would be more likely to fixate on the left operand of subtraction and division problems (i.e., the minuend and the dividend, respectively) when those operands are important in determining the nature of the required processing. For addition and multiplication, however, no specific problem element is more informative because operands are equally likely to be larger or smaller on the left and the right.

Additionally, we tested specific hypotheses that reflected predictions about processes that have been proposed in models of mental arithmetic. Following Zhou et al. (2012), we used eye-tracking data to assess whether solution processes were influenced by operand order for addition and multiplication problems. As described earlier, Chinese-educated participants solve multiplication problems where the smaller operand is first more quickly than the commutative pairs (e.g., 3 x 9 is solved faster than 9 x 3). This pattern is consistent with a model of mental multiplication proposed by Verguts and Fias (2005) in which participants store only one of the two commutative problems, accessing the solutions through a particular order of the operands. They suggested that the specific order was consistent for an individual, but might vary across individuals. Further, in the model of arithmetic proposed by Rickard (2005), both orders of the commuted pairs activate the same memory representation. Similarly, Butterworth, Zorzi, Girelli, and Jonckheere (2001) proposed a model of addition in which problems were stored in relation to the larger operand. Thus, if participants are using solution strategies that are tied to a particular operand order, these processes might be reflected in patterns of eye movements.

Method

Individuals solved problems from one of the four basic operations, that is, addition, subtraction, multiplication, or division. We used a between-groups design because there are documented effects of mixing operations beyond our current focus. For example, when operations are mixed within blocks, participants make more cross-operation errors such as 3 + 4 = 12 (Campbell & Arbuthnott, 2010; Miller & Paredes, 1990), show switch costs between operations (Campbell & Arbuthnott, 2010), and experience retrieval-induced forgetting (Campbell & Thompson, 2012).

Participants

One hundred and nine adults participated in this research. Table 1 presents the number of participants, the median ages, and gender distributions across each of the four operations. Nine participants (six in addition, one in subtraction, and two in division) were graduate students and were compensated with $10. The remaining participants were recruited from the undergraduate participant pool and were compensated with course credit in first- or second-year psychology courses. All participants reported normal or corrected-to-normal vision.

Table 1

Demographic Information About the Participants for Each of the Four Operations

Operation	*N (females)*	Age
Addition	30 (*12*)	19.5
Subtraction	20 (*7*)	20
Multiplication	30 (*18*)	21
Division	29 (*14*)	21

Note. Age is the median value in each group.

Materials

Addition and Subtraction

Addition problems consisted of all pairwise combinations of the operands between 2 and 9 including problems with repeated operands (e.g., 4 + 4). Commutative problems (e.g., 3 + 6 and 6 + 3) were considered unique, resulting in 64 different problems. Problems with a sum less than 10 were categorized as small problems, and problems with a sum equal to or greater than 10 were categorized as large problems. Subtraction problems consisted of the inverted set of addition problems where the left operand is replaced with the sum (e.g., for 6 + 9 and 9 + 6 the corresponding subtraction problems are 15 – 9 and 15 – 6). Accordingly, all large subtraction problems had a double-digit left operand.

Multiplication and Division

Multiplication problems consisted of all pairwise combinations of the operands between 2 and 9 including problems with repeated operands (e.g., 4 x 4). Commutative problems (e.g., 3 x 6 and 6 x 3) were considered unique.

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Problems with a product equal to or less than 25 were categorized as small problems and problems with a product greater than 25 were categorized as large problems. This categorization divides the problem sets into equal groups and is the typical way in which problem size is defined in the literature (e.g., Campbell & Xue, 2001; LeFevre, Bisanz, et al., 1996). Division problems consisted of the inverted set of multiplication problems where the left operand is replaced with the sum (e.g., for 6 x 9 and 9 x 6 the corresponding subtraction problems are 54 ÷ 9 and 54 ÷ 6).

Note that the problem-size categories do not completely divide the problems with shared operands in exactly the same way across operations: specifically, addition problems 2 + 9, 2 + 8, 3 + 7, and 3 + 8 are categorized as ‘large’ because they require a carry operation and have a double-digit sum (relevant for the subtraction set) whereas the multiplication problems 2 x 9, 2 x 8, 3 x 7, and 3 x 8 are categorized as ‘small’. Despite these differences, we used this definition of problem size to be consistent with previous research in order to provide valuable insight to future researchers using this standard set of problems.

Apparatus

We presented problems visually on a personal computer with a 15-inch monitor. Participants responded verbally into a headset microphone connected to an ASIO voice trigger accurate to +/- 1 ms. The experimenter recorded each response on a standard keyboard. We recorded eye movements using an EyeLink 1000 camera and software that measure the reflection of camera-emitted infrared lights from the participants’ corneas (SR Research). Recording refreshed at 500 Hz and the software’s parser parameters, which set thresholds determining events such as saccades and fixations, were set to the recommended values for cognitive research. As a result, a fixation was defined as a period of at least 100 ms in which the eye position does not move more than 0.15 degrees of visual angle.

Stimuli and Interest Areas

We presented problems in black 60 point Arial font on a white background. We defined three interest areas: one around each of the operands and another around the operator. Each interest area measured 7 cm by 7 cm and was centered on the stimulus component. We determined the size of the interest areas such that the borders of each interest area connected but did not overlap (see Figure 1). The single-digit problems subtended 14.25 degrees of visual angle. The problems with double-digit operands (large subtraction, most division problems) subtended 15.19 degrees of visual angle. We calculated total gaze duration as the sum of all fixation durations in each interest area per trial. Fixation counts were the total number of fixations in each interest area per trial.

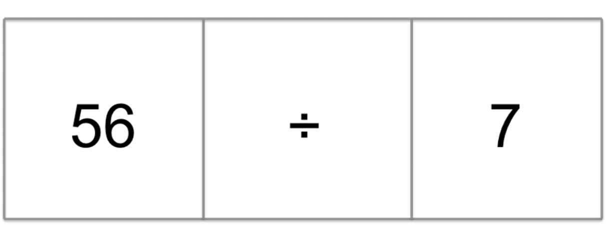

Figure 1. The three interest areas superimposed over an example problem.

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Procedure

Participants were tested individually in a quiet room. Participants initially read and signed an informed consent form. The eye-tracking camera, chin rest, and chair were adjusted such that the participants were comfortable and the camera provided a full view of the participants' eyes. The chin rest was 60 cm away from the computer monitor. Instructions on the computer screen indicated that the participants would be solving arithmetic problems verbally while their eye movements were recorded. Before starting the experiment, the eye tracker was calibrated. During calibration, nine circular points appeared in the four corners of the screen and the central perimeter points between corners in random order, and the participants were required to direct their gaze to each point. The next point appeared when a fixation was recorded. The experimenter evaluated the fixations and made a visual judgment regarding their accuracy. If the fixations were accurate, the experiment continued. Otherwise, the calibration process restarted. The camera was re-calibrated every 32 trials.

Participants were instructed to provide a solution to each problem as quickly and accurately as possible. At the start of each arithmetic trial, a 1.5 cm by 1 cm rectangular cue appeared in one of the four corners of the screen. The location of the cue varied randomly from one trial to the next. Participants were instructed to fixate on the cue when it appeared. The problem appeared when the computer detected a fixation on the cue. This procedure was used to ensure that participants were not gazing at the operator at the beginning of each trial, which might artificially inflate the number of fixations and total gaze duration to the central interest area (i.e., the operator). Each problem remained on the screen until the ASIO voice trigger had recorded a response. The screen remained blank until the experimenter had recorded the participant's spoken answer and initiated the next trial.

Participants first completed 10 practice trials to familiarize them with the experimental procedure. Subsequently, we presented each arithmetic problem three times in a pseudo-random order (i.e., problems were presented in three randomized blocks and problems did not repeat within blocks), resulting in 192 trials. After completing the experiment, participants were debriefed and dismissed.

Results

We discarded spoiled trials – in which the voice trigger either failed to register the first response or registered an extraneous sound (e.g., a sharp breath or an "umm") – from all analyses. This filtering resulted in the removal of 8.3% of addition trials, 5.4% of subtraction trials, 6.9% of multiplication trials, and 6.9% of division trials. We also removed trials on which the participants provided an incorrect response for the analyses of response times and eye-tracking measures. This filtering resulted in the removal of an additional 4.9% of addition trials, 1.8% of subtraction trials, 6.4% of multiplication trials, and 10.4% of division trials.

Response Time and Error Rates

Table 2 presents the mean of median response times and the mean error rates as a function of operation and problem size. Response times and error rates were analyzed in separate 4 (operation: addition, subtraction, multiplication, division) x 2 (problem size: small, large) analyses of variance with operation as a between-subjects factor and problem size as a within-subjects factor. Table 3 presents the relevant statistics.

Response times and error rates differed across operations and increased as a function of problem size. For small problems, error rates were low and very similar across operations, although higher in division than the other three

Table 2

Mean Solution Latencies and Percentage Errors by Problem Size and Operation

	Addition		Subtraction		Multiplication		Division	
Problem Size	**RT**	**%E**	**RT**	**%E**	**RT**	**%E**	**RT**	**%E**
Small	947	2.4	1046	2.6	1105	2.4	1145	5.5
Large	1241	7.1	1564	8.4	1408	10.8	1517	15.4
PSE	294	4.7	518	5.8	303	8.4	372	9.9

Note. PSE = difference between large and small.

Table 3

Statistical Analyses of Response Time and Error Rates

		Response Time			Error Rate		
Variables	***df***	***F***	***p***	η_p^2	***F***	***p***	η_p^2
Operation	3, 105	3.68*	.014	.095	8.06**	<.001	.187
Problem Size	1, 105	257.30**	<.001	.710	106.30**	<.001	.503
Operation x Problem Size	3, 105	4.32**	.007	.110	3.20*	.027	.084

*$p < .05$. **$p < .01$.

operations. Error rates were higher in large problems than in small problems for all operations, with the discrepancy (i.e., the problem size effect) largest in division and smallest on addition problems. These patterns of error rates are similar to those reported in other studies (e.g., Campbell & Xue, 2001) although the overall error rate is somewhat higher.

Participants responded more quickly to small than large problems. Differences between small and large problems (i.e., the problem size effect) were similar for addition and multiplication problems, somewhat greater on division problems, and largest on subtraction problems.

To evaluate whether the participants in the present research showed typical performance on simple arithmetic, these patterns were compared to those reported by Campbell and Xue (2001), which reports data on all four operations. The pattern of latencies for the non-Asian Canadians in that study (see their Table 2) is quite similar to that observed here, although overall latencies were slower in the present research. As in the present research, Campbell and Xue's participants responded fastest on small addition problems and slowest on large subtraction problems. Problem-size effects (i.e., differences between small and large problems; see Table 2) were quite similar to those in the present research for addition (283 ms), multiplication (351 ms), and division (339 ms). The problem-size effect was largest on subtraction problems (439 ms), as in the present research. Thus, compared to a similar population solving the same problems, the present sample showed somewhat slower and less accurate performance but quite similar patterns across the four operations. Hence, we conclude that, at least at a very gross level, participants' performance in the present study is comparable to that of similar studies.

Analyses of Eye Tracking Data

We extracted three dependent measures from the eye tracking data and analyzed each as a function of interest area: location of first fixation, total gaze duration, and total fixations. We assume that the first fixation location reflects an overall strategic orientation towards the specific operation, and thus is similar to the measure of horizontal eye movement analyzed by Zhou et al. (2012). Gaze duration and total fixations, in contrast, reflect the distribution of processing time across locations for each problem and thus we expect to see patterns of problem-size effects.

Location of First Fixation

We calculated the proportion of trials on which participants made their first fixation to each interest area. Because the proportions of first fixations are not independent across locations, we analyzed proportion of first fixations separately for each interest area in four (operation: addition, subtraction, multiplication, division) analyses of variance where operation is a between-subjects factor. Table 4 presents the relevant statistics[i]. For all operations, participants were most likely to first fixate on the operator (the centre interest area) and they rarely fixated first on the right operand.

Table 4

Separate Analyses of Proportion of First Fixations to Each Interest Area: The Independent Variable for Each Analysis is Operation

Interest Area	*df*	*F*	*p*	η_p^2
Left Operand	3,105	12.58**	<.001	.26
Operator	3,105	8.90**	<.001	.20
Right Operand	3,105	7.68**	<.001	.18

$^{**}p < .01$.

The effects of operation were significant for all three interest areas. Comparisons across operations were made using Bonferroni post-hoc tests (all *p*s < .05). For the left operand, the proportion of first fixations was significantly higher in division (.45) than in the other three operations, which did not differ (.30, .25, and .18 for subtraction, addition, and multiplication, respectively). For the centre location (i.e., the operation sign), proportion of first fixations was highest in multiplication (.76), significantly higher than subtraction (.53) and division (.51). Addition was not significantly different from the other operations (.65). For the right operand, the highest proportion of first fixations occurred in subtraction (.17), which was significantly different than multiplication (.05) or division (.04); addition was not significantly different from the other operations (.10). In summary, although participants were most likely to look first at the centre interest area, this trend was strongest for multiplication and addition; for division the first fixation was almost as likely to be to the left as to the centre operand; and for subtraction, a somewhat greater proportion of fixations occurred to the right operand. Because participants were only solving a single operation, these differences in the pattern of first fixations suggest that consistent patterns of initial eye fixations occur within an operation.

The total number of fixations to each interest area varied from a mean of about 0.5 to over 2.0, depending on operation and problem size. Thus, participants were able to process sufficient information about operands without fixating in every interest area on every trial. A first fixation to the centre interest area might often have allowed the individual to encode the left or the right operand without an actual fixation in that interest area.

Gaze Duration and Number of Fixations

Total gaze duration and number of fixations were analyzed as a function of interest area and problem size. Figures 2 and 3 present the mean gaze duration and mean number of fixations to each interest area as a function of operation and problem size. Gaze duration and number of fixations were analyzed in two separate 4 (operation: addition, subtraction, multiplication, and division) x 2 (problem size: small, large) x 3 (interest area: left operand, operator, right operand) mixed-model analyses of variance. Operation was a between-subjects factor, and problem size and interest area were within-subjects factors. Table 5 presents the relevant statistics. We interpreted interactions graphically using 95% confidence intervals as recommended by Masson and Loftus (2003; see also Jarmasz & Hollands, 2009).

As shown in Table 5, all main effects and interactions were significant for gaze duration and number of fixations. The three-way interactions are shown in Figures 2 and 3. The first interesting pattern is that, for both dependent measures, there are no problem-size effects for the centre interest area. Time and number of fixations in the centre interest area vary somewhat across operations, but in general, the absolute performance is relatively similar, suggesting that time and fixations in the centre interest area represent operation-independent processing. In contrast, gaze durations and fixations vary both with problem size and across operations for the left and right interest areas. Fixation patterns during addition and multiplication are similar (but not identical), and two unique patterns occurred during subtraction and division.

Table 5

Statistical Analyses of the Number of Fixations and Total Gaze Duration

		Total Gaze Duration			Number of Fixations		
Variables	***df***	***F***	***p***	η_p^2	***F***	***p***	η_p^2
Operation (Op)	3, 105	3.10*	.030	.081	9.20**	<.001	.208
Problem Size (PS)	1, 105	262.81**	<.001	.715	229.85**	<.001	.686
Interest Area (IA)	2, 210	7.70**	.001	.068	12.80**	<.001	.109
PS x IA	2, 210	36.88**	<.001	.260	64.82**	<.001	.382
Op x PS	3, 105	3.93*	.011	.101	6.52**	<.001	.157
Op x IA	6, 210	4.61**	<.001	.116	7.25**	<.001	.172
Op x PS x IA	6, 210	11.16**	<.001	.242	13.70**	<.001	.281

*$p < .05$. **$p < .01$.

The most striking pattern in the addition and multiplication figures is that, whereas gaze duration and number of fixations were greater during large than small problems, the overall fixation patterns were similar across problem size. This consistency was especially striking during multiplication. During the solution of both small and large problems, participants allocated approximately half of their fixation time to the operator and one-quarter to each operand. Fixation patterns were similar during addition except that participants allocated some additional fixation time to the right operand on large problems. Thus overall, fixation patterns are similar across addition and multiplication and are largely independent of problem size.

In contrast, patterns of fixation and gaze duration varied with problem size for both subtraction and division problems. While solving small subtraction problems, participants' fixation patterns were similar to those during addition and multiplication; they allocated approximately half their time to the operator and one-quarter to each operand.

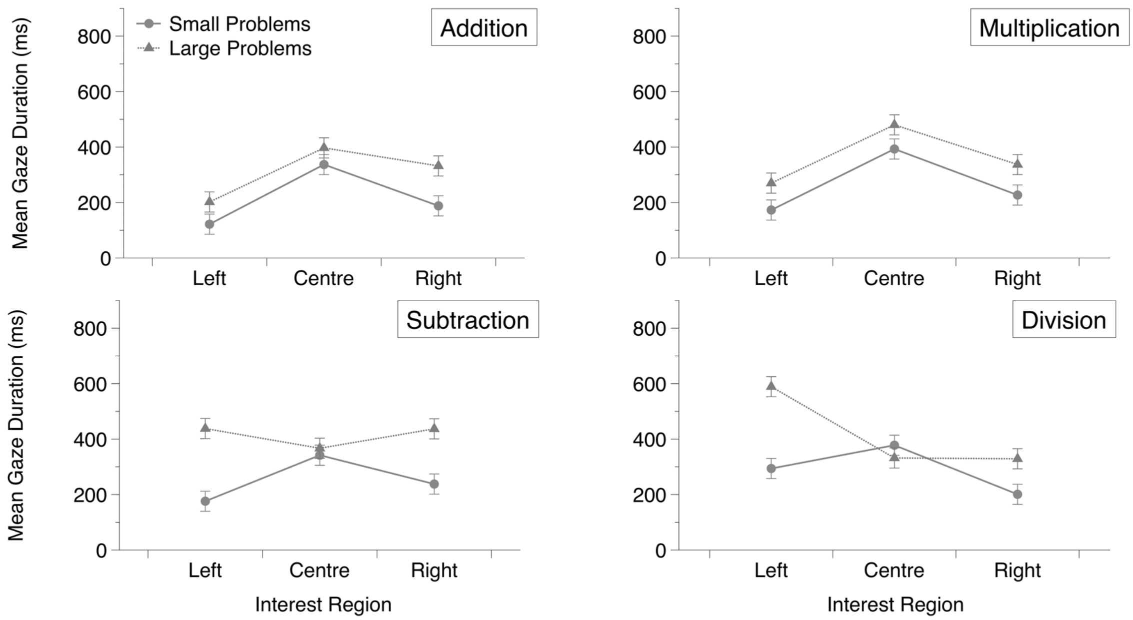

Figure 2. Mean gaze duration as a function of operation, problem size, and interest area. Error bars represent 95% confidence intervals.

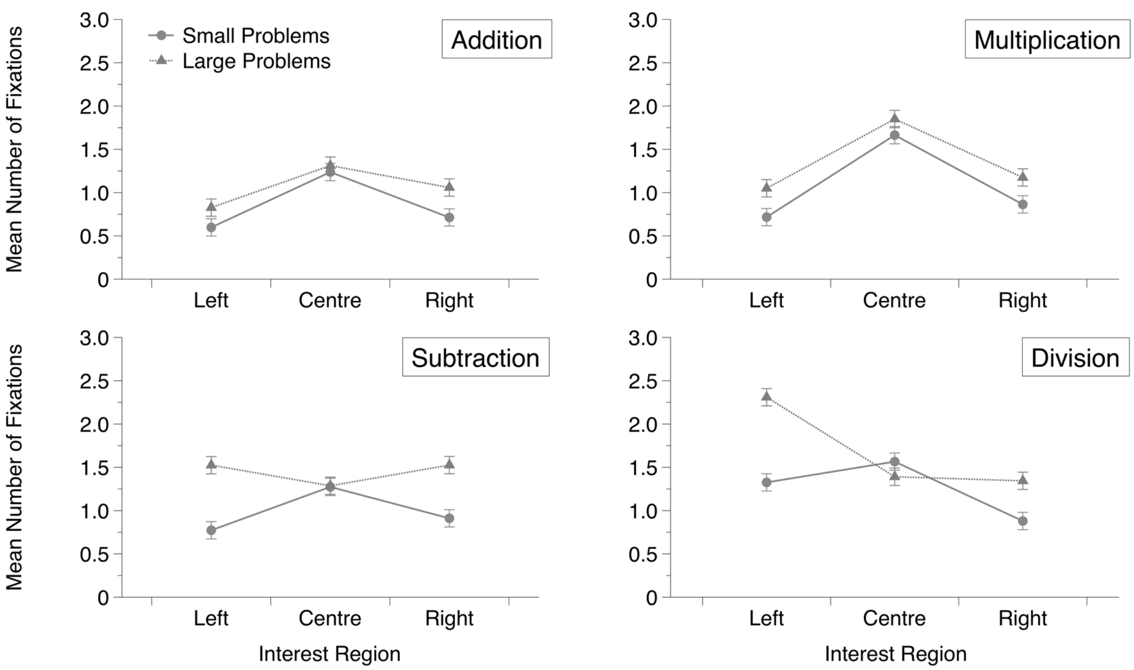

Figure 3. Mean number of fixations as a function of operation, problem size, and interest area. Error bars represent 95% confidence intervals.

However, during large subtraction problems, participants spent the majority of their time fixating on the operands and (relatively but not absolutely) less time on the operator. Fixation time was allocated equally between the operands.

Division problems elicited another unique fixation pattern. As with subtraction, fixation patterns during small problems were similar to the patterns during addition and multiplication although gaze durations and fixations were somewhat elevated for the left compared to the right operand. In conjunction with the first fixation results, this pattern might simply reflect the strategic bias during division to start processing at the left operand. In contrast, during large problems, participants allocated relatively more time to left operand compared to the centre and right operands, and much more than on any other operation. Thus, similar to subtraction, in division fixation patterns varied with problem size, but in contrast to subtraction, substantial additional time was spent on the left operand.

Fixation Patterns

To supplement the analyses of fixation counts and durations, and to facilitate comparisons of fixation patterns across operations and problem size, we divided the total time for each trial into six intervals. We then calculated the median fixation durations for each interval for each person. Figure 4 presents the mean of median durations across intervals for each operation and problem size. In contrast to our preceding figures, each interest area is now represented as a separate plot line rather than as three plot points.

Fixation patterns were very similar for the right interest area across all eight combinations of operation and problem size. Participants spent very little time fixating on the right operand in the first two intervals. In subsequent intervals, fixation time to the right interest area increased gradually, peaking between the third and fifth intervals and remaining stable thereafter. Furthermore, the absolute amount of time on the right interest area in final interval was quite similar across categories. Thus, fixation to the right interest area near the end of the trial might represent the conclusion of the processing sequence, similar to ending a sentence. In other respects, however, the distributions of processing time and locations was not at all similar to those found in reading.

The next most common pattern occurred on all addition and multiplication problems and on small subtraction. In the first two intervals, participants fixated on the centre interest area almost exclusively. For these problems, fixation time to the centre operand gradually decreased over subsequent intervals, shifting to the right operand. On small problems the centre remained dominant in the second half of the trial, whereas on large problems the times were similar in the centre and right interest areas. Very little time was spent on the left operand for addition, multiplication, or small subtraction. For these problems, therefore, participants fixated in the left interest area at some point during the trial (as shown in the mean fixation data), but not in a consistent pattern across individuals, resulting in similar and low mean fixations to the left operand across the six intervals. Accordingly, all addition, multiplication, and small subtraction problems showed a combination of centre-dominant plus right-shift patterns.

The processing patterns for large subtraction and for small and large division problems were distinct. For large subtraction problems, participants showed an initial bias towards the centre in the first interval, with equal processing time across areas in the second interval, a pattern that was moderated only slightly across the remaining intervals. Hence, processing time (after the initial interval) was thus roughly equivalently distributed across the three interest areas. This pattern might represent different patterns across individuals or specific items. Solution of large subtraction problems varies across individuals (LeFevre et al., 2006) and so further work that explores fixation patterns in relation to solution strategies will be necessary to understand how this processing pattern is linked to cognitive activities.

For small division problems, the main difference in processing patterns compared to other small problems was that in the first two intervals, participants fixated approximately equally to the left operand and the centre. This

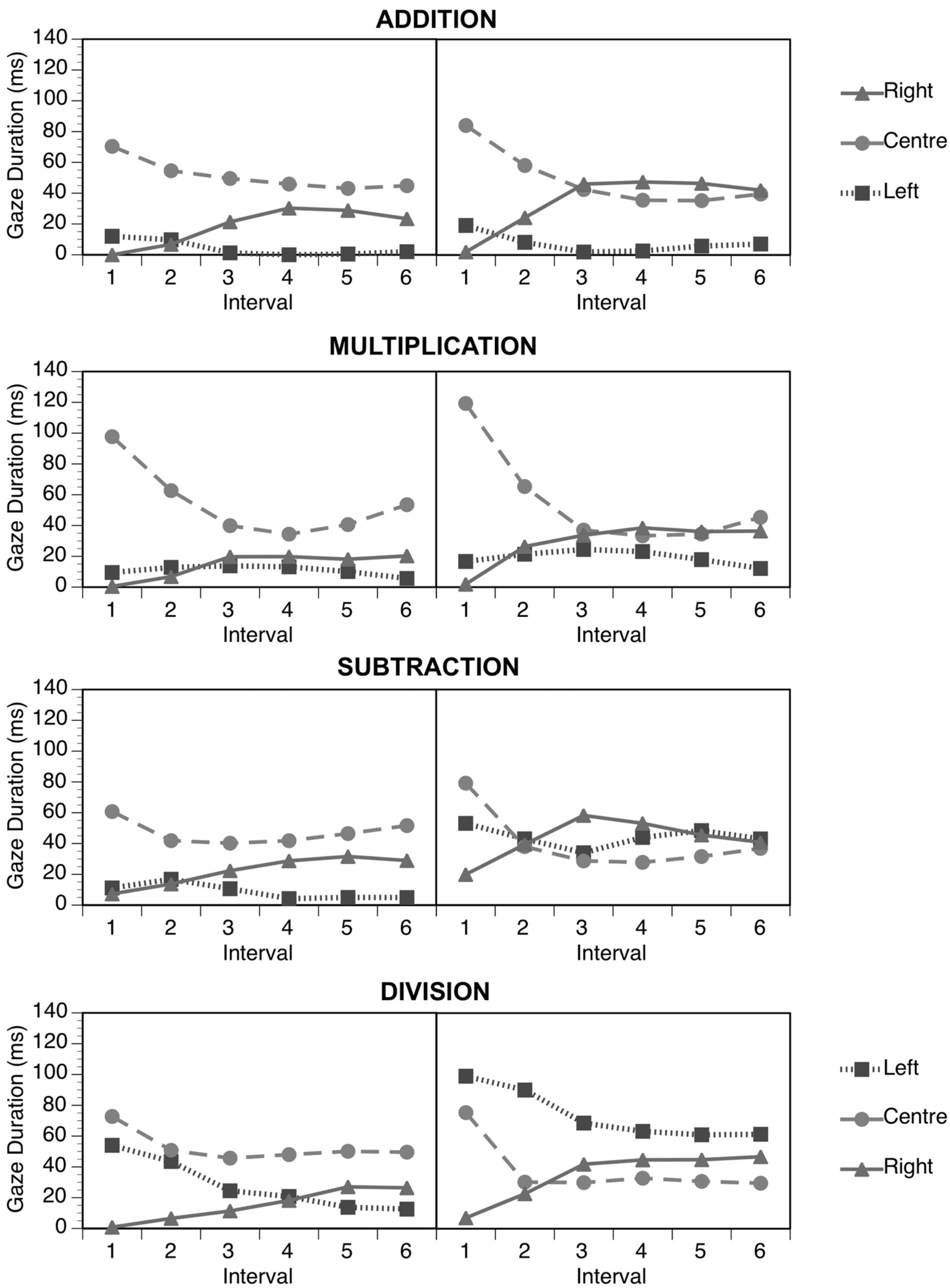

Figure 4. Means of median values of time-normalized gaze durations by operation and problem size for each interest area (see text for details).

pattern supports the view that processing time varies with the information value of the operands in that it was exacerbated for large division problems where the majority of time was directed to the left operand. Thus, small division problems were similar to other small problems, although processing patterns were much more strongly centre dominant for small addition, multiplication, and subtraction, whereas for small division, equal processing time was allocated to the centre and left interest areas in the initial intervals.

Large division problems were processed differently than the other seven categories, although the general right-shift pattern was still evident. Most of the fixation time was in the left interest area across the whole trial. Participants did look at the centre interest area relatively frequently in the first interval, but this dropped off quickly. Thus, the emphasis on the left operand, the large digit, was presumably not just an encoding effect but reflects the information value of the dividend to the problem solution. Large division can thus be characterized by left-dominant processing although it also showed the more general right-shift pattern.

In summary, there were similarities across operations and problem size in the progression of fixations, with common elements of early attention to the centre interest area and a gradual shift towards the right operand in the latter part of the trial. These patterns were moderated by a centre-dominant pattern for addition, multiplication, and small subtraction, early focus on the left operand for division problems, and finally by a left-dominant pattern for large division problems, where it was the focus of most of the processing time. In contrast to sentence processing, where a left-to-right processing pattern is nearly universal, these results suggest that processing patterns for arithmetic problems will vary substantially with problem characteristics.

Further Analyses

Residual Response Time

The total gaze duration – summed across the three interest areas – does not equal the total response time. In addition to fixating within the interest areas, participants also spend time making saccades from area to area, fixating outside of the interest areas, and blinking. To index the residual time not spent fixating on the three interest areas, we subtracted the summed gaze duration (GD) from the total response time and divided that difference by the total response time (RT). We then converted the proportion to a percentage, thus residual response time is equal to

$$100 * \frac{RT - GD}{RT}$$

Residual response times were analyzed in a 4 (operation: addition, subtraction, multiplication, division) x 2 (problem size: small, large) analysis of variance where operation is a between-subjects factor and problem-size is a within-subjects factor. Table 6 presents the relevant statistics. Residual response times did not significantly differ across operations (29.4%, 21.4%, 27.4%, and 25.5%, respectively). However, residual response times were significantly longer for small problems (28.9%) than for large problems (23.0%). There was no interaction between operation and problem size. Thus, the percentage of the total response time that is captured in the gaze duration measure is reasonably similar across problem categories.

Table 6

Statistical Analysis of Residual Response Time as the Percentage of Total Time

Variables	*df*	*F*	*p*	η_p^2
Operation	3, 105	2.00	.120	.054
Problem Size	1, 105	87.28**	<.001	.454
Operation x Problem Size	3, 105	0.25	.864	.007

**p < .01.

Operand Order

To evaluate effects of operand order for addition and multiplication problems, we analyzed gaze duration and number of fixations in two separate 2 (operation: addition, multiplication) x 2 (problem size: small, large) x 2 (operand order: left larger, right larger)[ii] x 3 (interest area: left operand, operator, right operand) analyses of variance where operation is a between-subjects factor and problem size, operand order, and interest area are within-subjects factors. Table 7 presents the relevant statistics. Consistent with the overall analyses, there were significant effects of problem size and interest area for both gaze duration and number of fixations, and an interaction of these variables. Only number of fixations showed a main effect of operation, however, with a mean of .98 fixations for addition compared to 1.24 for multiplication.

Table 7

Statistical Analyses of Number of Fixations and Total Gaze Duration for Addition and Multiplication, Including Operand Order as a Factor

		Total Gaze Duration			Number of Fixations		
Variables	***df***	***F***	***p***	η_p^2	***F***	***p***	η_p^2
Operation (Op)	1, 58	1.95	.168	.032	8.79**	.004	.132
Problem Size (PS)	1, 58	107.01**	<.001	.648	92.60**	<.001	.615
Operand Order (OO)	1, 58	3.10	.084	.051	7.68**	.007	.117
Interest Area (IA)	2, 116	19.72**	<.001	.254	29.62**	<.001	.338
Op x PS	1, 58	0.05	.831	.001	1.17	.284	.020
Op x OO	1, 58	0.88	.351	.015	0.12	.731	.002
Op x IA	2, 116	0.36	.716	.006	2.43	.093	.040
PS x OO	1, 58	2.67	.107	.044	0.69	.408	.012
PS x IA	2, 116	3.95*	.022	.064	8.54**	<.001	.128
OO x IA	2, 116	7.65**	.001	.117	13.59**	<.001	.190
Op x PS x OO	1, 58	2.37	.129	.039	0.35	.556	.006
Op x PS x IA	2, 116	1.08	.342	.018	0.99	.376	.017
Op x OO x IA	2, 116	0.17	.844	.003	0.47	.627	.008
PS x OO x IA	2, 116	0.14	.870	.002	3.93*	.022	.063
Op x PS x OO x IA	2, 116	0.26	.772	.004	2.24	.111	.037

*$p < .05$. **$p < .01$.

For both gaze duration and number of fixations, operand order significantly interacted with interest area. Participants spent more time processing larger than smaller operands when the larger operand was on the right (319 vs. 252 ms; 1.08 vs. 0.90 fixations) than when it was on the left (216 vs. 186 ms; 0.86 vs. 0.80 fixations). For number of fixations, the three-way interaction of operand order, interest area, and problem size occurred because this pattern was more pronounced for small than for large problems (see Figure 4). For small problems, the number of fixations on large operands on the left were greater than on other operands whereas on large problems, operands on the right were fixated more than those on the left, with the larger operand significantly higher than the smaller operand. Operand order did not interact with operation.

In summary, when the larger operand was on the right (e.g., 3 x 9 or 2 + 4), it attracted additional processing time. This pattern of results for operand order is consistent with models in which participants organize their cognitive networks in relation to the larger operand, as suggested by Verguts and Fias (2005) for multiplication and by Butterworth et al. (2001) for addition. On this view, smaller x larger problems might be slower to process because the larger operand needs to be identified before retrieval can be initiated. If participants can initiate retrieval automatically when the first operand is encountered, when the order is larger x smaller, processing will be facilitated. When the order is smaller x larger, in contrast, processing initiated from the left operand will need to be interrupted when the second operand is encoded, potentially accounting for the increased processing time on the right operand. On large problems, however, there was also an overall tendency to fixate more on the right than on the left operand.

Double-Digit Operands

We hypothesized that participants might spend more time processing double- than single-digit operands, based on data demonstrating that double-digit numbers are less frequent than single-digit numbers and take longer to name (Brysbaert, 1995). Hence, additional encoding time might be needed when the operand is a larger number. Because subtraction and division problems include double-digit operands, we might expect some of the additional processing time on these problems to be directly linked to the larger operand. Accordingly, we examined processing time on double-digit operands within each operation.

Fixation patterns during subtraction problems directly contradict the double-digit hypothesis. During large subtraction problems, which, by definition, contain a double-digit left operand and a single-digit right operand, fixation time was evenly distributed between the operands. Participants did not allocate more time to the double-digit operand than to the single-digit operand.

Evaluating the hypothesis for division problems was less straightforward, because problem size is not perfectly indicative of single- versus double-digit left operands. Thus, to evaluate the double-digit proposition in the context of division, we conducted two separate 2 (operand: left, right) x 2 (left operand digits: single, double) repeated-measures analyses of variance on gaze duration and number of fixations. The double-digit hypothesis would be supported by a significant interaction in which the difference between fixations to the left and right operands is greater when the left operand has double digits. The interaction terms were significant in both analyses ($ps < .001$). When the left operand was a double-digit number, participants fixated on the left operand for 185 ms longer (0.79 more fixations) than on the right operand. In contrast, when the left operand was a single-digit number, participants only fixated on the left operand 6 ms longer (0.16 more fixations) than on the right operand.

This analysis should, however, be taken lightly. The majority of division problems have a double-digit left operand (90.6%). Additionally, the number of digits in the left operand is entirely confounded with problem size such that the easiest (i.e., fastest and least error prone) division problems have single-digit left operands. Given these limitations, more definitive tests of the double-digit hypothesis are needed with more varied stimuli.

Discussion

Participants solved addition, subtraction, multiplication, and division problems while their eye movements were recorded. We analyzed the total gaze duration and the number of fixations to the left operand, the operator, and the right operand. We also reported the percentage of trials in which participants made their first fixation on each of those interest areas. We expected that the large and reliable differences in response time and error rates associated with problem size would also be reflected in eye movements. These expectations were partially confirmed. However, we also found differences in fixation patterns among the four operations. Specifically, we found three unique patterns of eye movements: one pattern for addition problems, multiplication problems, small subtraction problems, and small division problems, a second pattern for large subtraction problems, and a third pattern for large division problems.

The existing literature on simple arithmetic problem solving has provided a wealth of details on how these problems are processed, based on response time, error analyses, and self-reports (Ashcraft & Guillaume, 2009). Very few studies, however, have explored the online processing that occurs during solution. Theorists generally agree that arithmetic computations include encoding, calculation processes, and answer production processes (e.g., Campbell, 1999). Each of these processing stages might be influenced by the characteristics of the problem. Thus, despite the superficial similarity of the four operations and the lack of detailed information provided by standard response time analyses, it is reasonable to assume that the allocation of cognitive processing activities varies across operations and problem size.

Addition and Multiplication

Our data support the view that addition and multiplication problems are processed similarly. We conclude that initial processing of these simple problems involves one relatively short fixation on each operand, typically after a first fixation to the centre of the display. In some cases, the first fixation in the centre might also allow the participant to identify one of the operands, and so not all interest areas are fixated on every trial. After encoding the operands, about half of the total processing time was allocated to the centre interest area, and approximately one-quarter to each of the operands. Presumably, the time allocated to the centre interest area after the initial interval reflects calculation processes (i.e., cognitive activities involved in producing an answer). This inference is consistent with increased processing time to the right operand in the course of the trial. The main difference in eye movements across problem size was slightly increased processing time on the operands for large problems. Given the predictability of the operation on each trial (i.e., each participant only experienced a single operation), and the uniformity of the operands (i.e., single-digit numbers from 2 to 9), the pattern of eye movements on addition and multiplication presumably reflect the least complex processing events for arithmetic problems.

The similarity in patterns of processing observed for addition and multiplication is consistent with results of other research, especially when the calculation phase comprises memory retrieval (Miller, Perlmutter, & Keating, 1984). Much of the processing time beyond initial encoding appears to reflect internal cognitive activity, with little time allocated to differential encoding or refreshing of the numbers in working memory during the course of problem solution.

Fixation patterns during addition and multiplication were also influenced by operand order. Participants allocated extra fixation time to the larger operand, but particularly when the larger operand was on the right. This pattern can be interpreted as evidence in favour of a mental representation in which problems are stored and activated in relation to the larger operand (Butterworth et al., 2001; Verguts & Fias, 2005; cf. Robert & Campbell, 2008). If activation is initiated automatically when the first operand is encoded, then the additional time on the second operand when it is larger might reflect an interruption or redirection of processing. However, this effect was small and other research with Canadian participants has not shown evidence for order-specific arithmetic representations (LeFevre & Liu, 1997; Robert & Campbell, 2008). Thus, further research with individuals whose educational histories are consistent with order-specific representations is necessary to fully understand this pattern.

There are also differences in how addition and multiplication problems are solved. Whereas addition and multiplication are usually solved via retrieval from memory (compared to subtraction and division), retrieval is used more often on multiplication than addition (Campbell & Xue, 2001; LeFevre, Bisanz, et al., 1996; LeFevre, Sadesky, & Bisanz, 1996). In both cases, the most frequent alternative to retrieval is use of some related fact (e.g., solving 7 + 8 as 7 + 7 + 1), but participants report counting on addition problems more frequently than on multiplication problems. These differences in retrieval efficiency between addition and multiplication might be greater in less-skilled samples, resulting in more similar problem-size effects across addition and multiplication in more recent studies compared to older studies (cf. Campbell & Xue, 2001; Miller, Perlmutter, & Keating, 1984).

More generally, other research suggests that addition and multiplication have similar mental representations. Both operations show effects of obligatory activation, the activation of arithmetic relations even when arithmetic is not required (Galfano, Rusconi, & Umiltà, 2003; LeFevre, Bisanz, & Mrkonjic, 1988), suggesting that the operands form a mental chunk. The identical elements model also assumes that addition and multiplication problems are similarly represented (Rickard, 2005), for example, with commutative problems being represented as a single unit in memory (e.g., 3 + 4 and 4 + 3). Thus, the finding in the present research that addition and multiplication show similar eye-tracking patterns is consistent with results of various other paradigms where these operations are shown to be similar. Our data, with the pattern of relatively more time on the centre interest area than on the operands, suggest that most of the memory and calculation processing, as well as answer production, occurs internally and thus is not strongly linked to specific locations, that additional fixations on operands are rarely needed to refresh operand identities, and thus that direct activation from the operands to the memory and calculation processes occurs relatively quickly. In summary, overall patterns of processing were relatively similar in addition and multiplication.

Subtraction and Division

In contrast to the results for addition and multiplication, subtraction and division provided showed much more variable patterns of processing. This variability can be linked both to operand characteristics, specifically information value, and to known differences in processing across these operations. On small subtraction problems, fixation patterns were very similar to those during addition and multiplication. Participants spent much more time fixating on the operator, approximately 45% of the total time, than on either of the operands. All operands are single digits (e.g., 7 – 5; 4 – 2) and solutions are generally achieved through memory retrieval (LeFevre et al., 2006). In contrast, on large subtraction problems, participants spent approximately 30% of their time fixating on the operator, with the remaining time distributed approximately equally across the operands. The additional time spent fixating on the operands (compared to the operator) is the opposite of the pattern observed during addition and multiplication. First fixations were most frequent to the operator but at a lower percentage (50%) compared to addition and multiplication, with a correspondingly higher percentage of first fixations to the left operand.

Participants are most likely to report solutions other than memory retrieval for large subtraction problems (compared to all other operations and problem categories) and it typically elicits the largest problem size effects of the four operations (Campbell & Xue, 2001; LeFevre et al., 2006). Subtraction problems might be reformulated as addition (e.g., 14 - 6 as 6 + ? = 14; Campbell, 2008). Mental transformations might involve more or longer fixations on the operands if alternative arrangements of the digits are activated and maintained in working memory. The very different distribution of total fixations and patterns of processing on large subtraction problems, as compared to addition, multiplication, and small subtraction, suggests that processing of the operands reflects both encoding and calculation processes.

The pattern of processing for small division was more similar to subtraction than to addition and multiplication. Initial fixations were approximately equally distributed to the centre and left interest areas, suggesting that a strong left-first processing bias occurred in division. Fixation patterns were also moderately similar as those in subtraction in that on small problems, fixations numbers and durations were similar across interest areas for small problems. However, processing patterns on large division problems were different from those on other operations. Participants made more fixations and spent substantially more time on the left operand during large division problems than during any other type of problem.

Both subtraction and division are sometimes solved via access to the corresponding operation (Campbell, 2008; Mauro, LeFevre, & Morris, 2003), but subtraction is also solved by counting and other transformation procedures (LeFevre et al., 2006). Division problems might access memory for multiplication facts routinely, even automatically, rather than activating a separate division memory representation. However, the finding that the eye tracking patterns are quite different between division and multiplication suggests that the access is mediated differently, through the “answer” (left operand) on division problems, rather than through the operands on multiplication problems (Rickard, 2005). In contrast, one source of the extra time on the operands for subtraction could be counting processes. These would be distributed across the operands, because counting could involve either counting up from the subtrahend or counting down from the minuend, depending on which counting strategy that the participant favored on a given trial.

The variability across operations that we observed for fixation patterns and durations is generally consistent with the view that, in contrast to small problems and all addition and multiplication problems (Campbell & Xue, 2001; LeFevre, Bisanz, et al., 1996; LeFevre, Sadesky, & Bisanz, 1996), large subtraction and large division problems are less likely to be solved via direct retrieval from memory (Campbell & Xue, 2001; LeFevre et al., 2006; LeFevre & Morris, 1999). During problems solved by direct retrieval, we expect that the operands are only the targets of fixation to the extent that they must be initially encoded. We speculate that once the operands are encoded, participants stop extracting information from the screen in order to retrieve the solution. On this view, the patterns of fixations on small problems suggest that participants simply fixate on the most convenient location, straight ahead to the center of the screen (i.e., on the operator). In contrast, alternative strategies involve multiple steps and take more time. We speculate that, depending on the exact strategy, additional information must be extracted from the operands, and, as a result, fixation times to the relevant operand or operands increase at each step.

It must be noted that, while our account fit our data, it is a post hoc explanation and so further empirical work is required to evaluate our hypothesis linking processing patterns to specific solution approaches. More work is also needed to determine how closely eye-tracking patterns are associated with specific solutions across different types of problems. Moeller et al. (2011) compared participants’ eye movements to carry and no-carry problems (e.g., 34 + 29 = 63). They found that participants looked more at the unit digits of the second operand and at the decade digit of presented answers on carry than on no-carry problems, supporting the view that eye movement patterns are sensitive to problem characteristics and to solution strategies.

Information Value of Operands Versus the Operation Sign

Operands carry more information value than operation signs in the present design because each participant saw only one of the four operations. Given that the information value of the stimuli seems important, why do participants make their first fixation to the least informative component of the problem? Because participants had to move their eyes from a corner of the screen to the centre when the problem appeared, it might be that the centre location

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is simply the most likely target of such an eye movement. Similarly, Schneider et al. (2012) found that participants look slightly to the left of centre when processing equations such as (((3+2)+4)+1) or (3+(2+(4+1))), moving to the starting set of parentheses (left or right, respectively) after the initial orienting fixation. In the present research, initial fixations tended to be to the centre interest area, with further fixations concentrated on the more relevant portions of the equation. Participants in the present research might have been able to gain enough information about one of the operands from an initial centre fixation and thus minimized overall eye movements. Further research in which the identity of the operation sign varies is necessary to further explore reasons why the centre of the screen is the preferred fixation location.

For division problems, participants presumably allocated more time to the left operand during division because it provides more information than the right operand. In addition and multiplication, as already discussed, the operands may provide somewhat different information, but the effects are very small (i.e., operand order). In contrast, the left and right operands provide approximately equal information during subtraction. Consider the set of subtraction problems in which the left operand is 11. In the standard set of problems, the solution can be 9, 8, 7, 6, 5, 4, 3, or 2. Participants are obligated to identify the right operand in order to solve the problem. In contrast, consider the set of division problems in which the left operand is 56. The only possible solutions are 7 and 8; the left operand greatly restricts the number of candidate responses. From the entire set of large subtraction problems, each left operand corresponds to between one and eight solutions with a mean of 4.78. In contrast, from the entire set of large division problems, most of the left operands correspond to two possible solutions and a smaller set of left operands only correspond to one possible solution. Thus participants seem to allocate more fixation time to the most informative operand.

Double- Versus Single-Digit Operands

We found little support for the prediction that participants would spend more time fixating on double-digit than single-digit numbers in subtraction (e.g., 12 - 3). Although Brysbaert (1995) found that people fixate longer on larger than on smaller numbers in naming and identification tasks, the extra amount of time was quite small relative to the total processing time on arithmetic problems. For division problems, the prediction about double-digit operands was difficult to evaluate because a very limited set of division problems include a single-digit left operand and because operand size and problem size are inherently confounded. We propose that the standard set of arithmetic problems used in our experiment is not appropriate for testing the double-digit prediction. However, the issue remains important for understanding the processes involved in solving problems such as 42 + 9, for example. In summary, patterns of processing across operation and problem size can be linked to the information value of the specific operands during each operation.

Conclusions

In conclusion, the present research was the first to explore fixation patterns during the standard set of simple arithmetic problems across the four arithmetic operations. Fixation patterns differed as a function of both operation and problem size, and the influence of problem size differed among operations. We propose, albeit speculatively, that the differences among operations reflect several factors, including differential use of non-retrieval solution strategies, differential attention to more informative problem components (e.g., for the dividend in division), and problem characteristics, such as problem size and operand order, that might influence solution processes across operations. Our data provide the foundation of a database of fixation patterns during mental arithmetic across operations and problem size, and open up avenues for further research utilizing eye-tracking methods to understand the cognitive processes underlying mathematical cognition.

Notes

i) We did not test effects of problem size because participants would not be able to predict problem size before the trial started.
ii) We excluded problems with repeated operands (e.g., 4 x 4) from the analyses.

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Competing Interests

The authors have declared that no competing interests exist.

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