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Commentaries

Mathematical Competence, Teaching, and Learning

Reflections on 'Challenges in Mathematical Cognition' by Alcock et al. (2016)

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Abstract

In this commentary, I focus on the notion of competence and issues related to the distinction between knowing how mathematical problems are solved versus knowing how to teach mathematics. Although definitions of competence may necessarily be affected by value judgements and thus less amenable to factual answers, providing a defensible definition is important because it affects eligibility for intervention and treatment. One way to tackle this issue is to focus on the identification of prerequisite skills and concepts needed for particular domains of mathematics. Recent work on fraction and algebra has shown that long held assumptions may need to be re-examined. On knowledge versus application, some cautionary notes are made on the importance of not losing sight of translating our knowledge of processes involved in mathematical problem solving into better pedagogical practices. [Commentary on: Alcock, L., Ansari, D., Batchelor, S., Bisson, M.-J., De Smedt, B., Gilmore, C., . . . Weber, K. (2016). Challenges in mathematical cognition: A collaboratively-derived research agenda. Journal of Numerical Cognition, 2, 20-41. doi:10.5964/jnc.v2i1.10]

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Laying out a research agenda is seldom an easy task; the more so when there are multiple investigators and when one is aiming at developing an agenda for the whole field. I applaud the authors for putting together a thoughtful and useful set of questions. My comments are focused largely on the notion of competence and the distinction between knowing how mathematical problems are solved versus knowing how to teach mathematics.

Competence Development

In the section on mapping predictors and processes of competence development (Alcock et al., 2016, this issue), one question that deserves some attention is how we go about defining competence. Although this question may fail the authors' selection and refinement criteria on the grounds that it may not have a factual answer that does not depend on value judgements, it is important because our definition of competence affects directly whether individuals' achievements are considered adequate. Take, for example, children who perform poorly from countries that rank at the top of international comparisons of mathematics achievement. Many of these children perform at levels close to the mean when considered with their international peers (Mullis, Martin, Foy, & Arora, 2012). A country specific norm-referenced definition of competence, together with likely concerns that these children are nonetheless performing more poorly than their peers in the same educational system, will likely direct intervention efforts to them. In contrast, adopting a criterion-referenced definition of expected competence, derived from international norms, will likely classify such children as possessing skills similar to their typically developing peers elsewhere.

The issue of criterion- versus norm- referenced definition is pertinent to Questions 5 to 9. Questions 6, for example, asks “What are the key mathematical concepts and skills that children should have in place prior to the start of compulsory education?” Ostensibly, this question is concerned with a criterion-based assessment of competence. It may have attracted a criterion approach because of the sense that early mathematical concepts, developed prior to formal schooling, are of such a fundamental nature that children either develop these skills to a satisfactory level or they do not. Questions 7 to 9, on relations between early numeracy and other aspects of mathematics, seem to focus more on a norm-referenced view of competence and are focused on identifying factors that contribute to individual differences in competence.

Given the hierarchical nature of mathematics, the question of the kind of mathematical concepts and skills that should be in place prior to the introduction of higher order concepts can and should be posed for each domain of mathematics in the K-12 curriculum. In evaluating this suggestion, it should be noted that notions of hierarchy in mathematical concepts is not uncontroversial. With respect to fraction knowledge, there are well established difficulties associated with the over-generalisation of whole number concepts to fractions (Ni & Zhou, 2005), and it has been proposed that whole numbers are biologically primary and fractions, at least more complex fractions and their operations, are biologically secondary (Geary, 2006). One curriculum implication that arises from these views is that children should be allowed to reach a certain level of competency with whole numbers before advancing to fractions. On the other hand, Siegler, Thompson, and Schneider (2011) argued that learning about fractions is part of a continuous process in numerical development, in which children need to learn about numerical magnitudes and the defining characteristics of each class of numbers. If this is the case, there seems to be no *a priori* reasons for placing the teaching of whole numbers and fractions sequentially.

A similar debate concerns algebra. Many curricula are arranged based on the belief that a foundation in arithmetic is required before the introduction of algebra. However, there is now a growing literature that shows that even children in primary school exhibit algebraic thoughts (Kaput, Carraher, & Blanton, 2008; Lee, Khng, Ng, & Ng Lan Kong, 2013; Ng & Lee, 2008) and can be successful in solving algebraic problems (Fuchs et al., 2008; Lee, Ng, & Ng, 2009). Indeed, there are suggestions that introducing tasks that require algebraic thinking early may overcome problems often encountered by children that learn algebra after a solid grounding in arithmetic (Kaput et al., 2008).

These observations suggest that amongst typically developing children at least, the relation between early acquired and later acquired, more complex mathematical skills may be weaker and less hierarchical than previously thought. However, whether strict hierarchy or necessary prerequisites exist for certain domains of mathematics seem to be questions that can be answered empirically and thus are not entirely dependent on value judgements. In contrast, questions regarding timing of acquisition of specific skills, though possibly related to periods of sensitivity (as found in the acquisition of language), seem more dependent on societal expectations regarding the kind of problems children of particular ages can be expected to solve.

The question of necessary skills and concepts is of particular importance for children with developmental difficulties. At present, diagnoses are often made on the bases of mathematical performance that is poorer than would be expected from the child's general intellectual abilities. However, even here, judgement of competence will likely be influenced by ideological beliefs regarding curricular orientation. As Geary (2004) argued, systems that emphasize conceptual understanding tend to place less importance on procedural fluency, with deficits in arithmetic fact retrieval not considered a serious concern.

With reference to atypical development, Geary (2004) also noted that regardless of whether children's difficulties are classified as belonging to the procedural, semantic, or visuospatial subtype, there are notable neuropsychological features associated with each subtype. Thus, although the influences of education and early experiences should not be underestimated, biological influences are also important, especially for children with more entrenched difficulties in mathematics. Given the utility of cognitive neuroscience in this regard, it is a pity that questions related to cognitive neuroscience did not make it through the winnowing process.

Knowing how Mathematical Problems Are Solved Versus Knowing how to Teach Mathematics

To help organise my thoughts regarding the overall structure of the questions, I took the liberty of putting them on a concept map (see Figure 1). I classified the five major areas into two major domains: those that fall under teaching versus those that focus more on discovering how students and mathematicians learn and solve mathematical problems. As illustrated in Figure 1, I have also added connectors between issues in teaching and learning that seem most closely related. Notably, there are a number of issues in the learning domain that are difficult to link to teaching and intervention. The lack of such explicit linkages may be due to a genuine mismatch; in that issues most critical in each domain are at different stages of discovery (e.g., we know how a specific domain of mathematics is processed by expert problem solvers, but do not know how students attain the same level of processing proficiency). Nonetheless, as our ultimate goal is to help students achieve an optimal level of mathematical proficiency, it seems important that insights from knowing how problems are solved have counterparts in translating that knowledge into better pedagogical or curricular practices.

With respect to promoting both procedural fluency and conceptual understanding, Alcock et al. (2016, this issue) argued that it would be more fruitful to focus not on which orientation is superior, but how combinations of the two can be effected to enhance learning. Here, it may be useful to consider whether different aspects of learning are likely to be affected by different instructional orientation. If a conceptual focus, for example, improves learning by "providing mathematical learning situations that are personally meaningful to the student", it would seem that its efficacy is effected primarily via non-cognitive factors. This brings us to the reasons for advocating an emphasis on conceptual, procedural, or a combination of the two types of knowledge in mathematical education. On whether a particular type of orientation produces superior outcomes, should the focus primarily be on achievement scores or are outcomes, such as increased motivation to learn, of equal importance? In this context, it may be the case that different orientations have different effects on different criterion measures. Without wanting to reignite debates regarding the superiority of the different approaches, perhaps the question we should focus on is not so much whether one or a combination of approaches is categorically superior to the other, but a more nuanced examination of the suitability of each orientation to specified goals: be they to improve motivation for learning, to achieve automaticity, or to fulfil societal needs for particular kinds of competency.

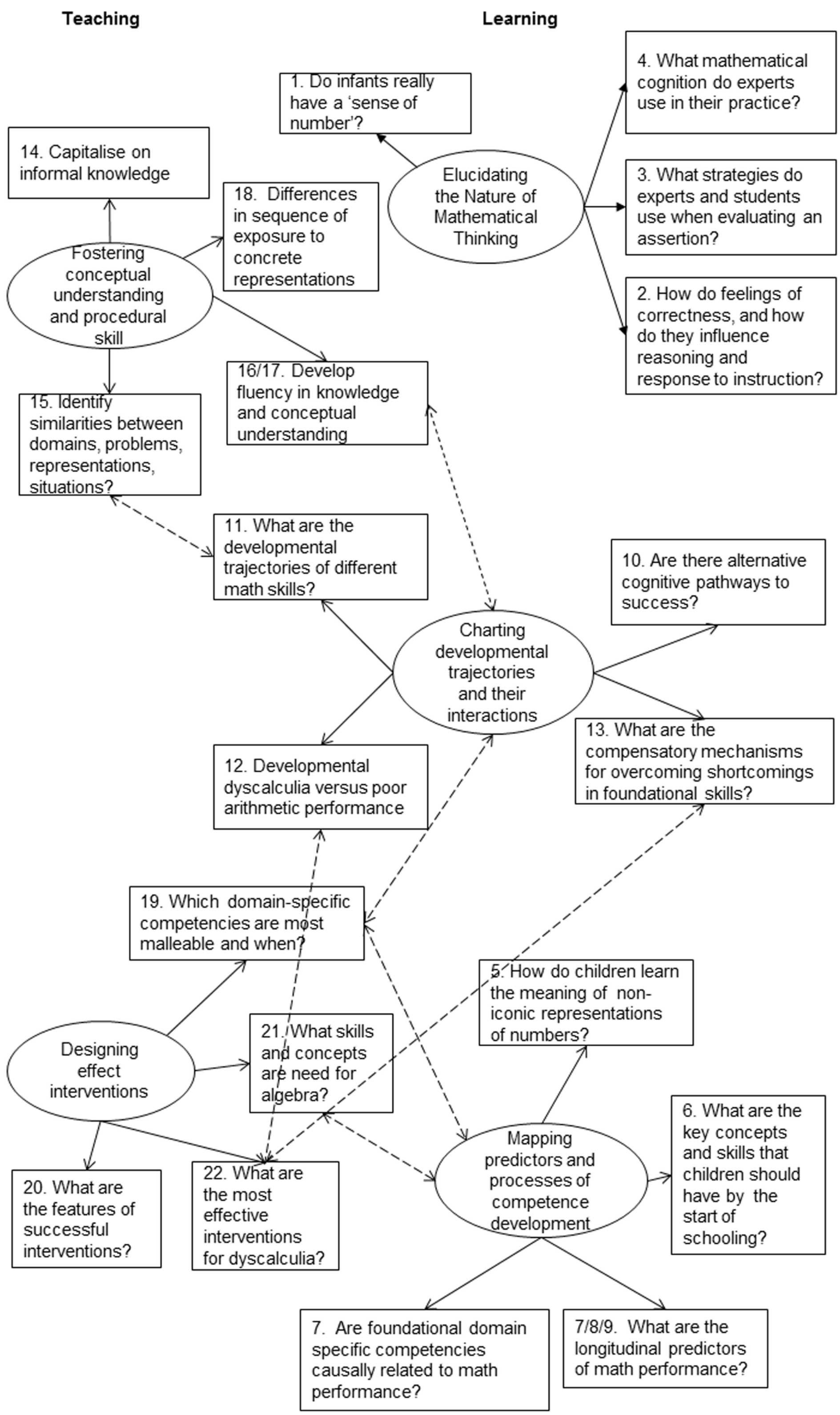

Figure 1. Domain areas and questions identified in "Challenges in mathematical cognition: A collaboratively-derived research agenda". Dotted lines denote potential interconnections between questions and domain areas.

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