Developing a Rigorous Measure of the Pre-school Home Mathematics Environment

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Abstract

Children begin pre-school with varying levels of school readiness. Those children who enter pre-school with better foundational mathematics skills are more likely to succeed in school than those who do not. This initial variation in early mathematics suggests that experiences outside of the school setting, namely the home environment, may support learning and development. This study aims to systematically develop a comprehensive home mathematics environment questionnaire that reliably assesses the experiences of pre-school children (i.e. 3-5-year-olds) following recent recognised scale development and validation methods. Four studies were used to develop and validate the Pre-school Home Mathematics Questionnaire (PHMQ). Study 1 focused on (a) item generation through individual, in-depth interviews with parents of young children and (b) identifying previous questions from other home mathematics environment (HME) questionnaires to be incorporated into the PHMQ. Study 2 involved questionnaire refinement and was used to assess the psychometric properties of the new measure while addressing construct validity (i.e. factor structure and scale score reliability). Study 3 assessed content and criterion validity of the scale. Finally, Study 4 focused on construct validity through confirmatory factor analysis. Overall, the four studies demonstrate construct, content, and criterion validity. Hence, the newly developed PHMQ satisfies the American Psychological Association (APA) standards for psychometric adequacy.

Keywords: (Max 7): pre-school, questionnaire, home numeracy environment, home mathematics environment, construct validity, content validity, criterion validity

Introduction

Children begin pre-school with varying levels of school readiness, with those children who enter pre-school with foundational mathematics and reading skills more likely to succeed in school than those who do not (Duncan et al., 2007; Watts, Duncan, Siegler, & Davis-Kean, 2014). This initial variation in early mathematics and reading skills suggests that experiences outside of the school setting, namely the home environment, may support learning and development (Manolitsis, Georgiou & Tziraki, 2013; Pomerantz, Moorman & Litwack, 2007). Research indicates that the home learning environment (i.e. the home literacy and numeracy environment) is a significant predictor of reading and mathematics achievement (Anders et al, 2012; Melhuish et al., 2008) but also can more broadly influence children's social and behavioural development (Sylva, Melhuish, Sammons, Siraj-Blatchford & Taggart, 2004; 2008). School-entry mathematical skills were found to be more important in predicting later mathematical, reading and science achievement than school-entry reading skills (Claessens & Engel, 2013), highlighting the importance of early mathematical knowledge in school readiness development. Therefore, it is essential to understand how early mathematics skills develop due to its impact on academic achievement more generally.

The home learning environment

The frequency of home learning activities has been established to have impact on child development. For example, Melhuish et al. (2013) investigated the long-term effects of different pre-school provision on child development and found that children from homes with the lowest frequency of home learning environment activities were almost 3 times less likely to attain Level 5 in mathematics at the end of Key Stage 2 (i.e., 11-year-olds in Northern Ireland), than children from homes with a higher frequency of home learning environment activities. Thus, it has been suggested that the frequency of home learning environment activities can diminish or benefit individual success later in life (Sénéchal & LeFevre, 2002).

Studies that explore the nature of the home learning environment have found wide variations between families. For instance, the quality of the home learning environment is associated with the availability of educational resources, for example books and board games (Anders et al., 2012; Cankaya & LeFevre, 2016; Gunn, Simmons & Kameenui, 1995; Melhuish et al., 2008; Skwarchuk, Sowinski & LeFevre, 2014). Previous research demonstrates that the quality of the home learning environment can be investigated either in relation to the domain of home literacy or home numeracy (Huntsinger, Jose, Liaw & Ching, 1997; Huntsinger, Jose, Larson, Balsink Krieg & Shaligram, 2000; LeFevre et al., 2009; Sénéchal & LeFevre, 2002) or irrespective of domain (Anders et al., 2012; Melhuish et al., 2008). There is a vast amount of literature that examines the role of the home literacy environment in contrast to the emergent literature on the home numeracy environment (HNE; Burgess, Hecht & Lonigan, 2002; Frijters, Barron & Brunello, 2000; Hart, Ganley & Purpura, 2016; Kirby & Hogan, 2008; Sénéchal & LeFevre, 2002; Sénéchal, LeFevre, Thomas & Daley, 1998). Hence, more research is necessary to understand the impact of the HNE.

The motivation for the creation of home mathematics environment (HME) measures has been grounded in evidence that since the early home environment (i.e. during pre-school years) has been connected to children's literacy skills it is theoretically reasonable to predict that the early home environment will impact children's numeracy skills (Blevins-Knabe, 2016; LeFevre et al., 2009; 2010; Lukie, Skwarchuk, LeFevre & Sowinski, 2014). Accordingly, researchers have adapted questions from home literacy environment questionnaires or generated novel questions to create HME questionnaires. Alternatively, other home numeracy questionnaire measures have been based on variations of the Home Observation for Measurement in the Environment (HOME) inventory (Caldwell & Bradley, 1984) for example, Anders et al. (2012). However, as the development of measurements should ideally be both deductive and inductive (Williamson, Karp, Dalphin & Gray, 1982) the current study used both rigorous approaches for questionnaire development.

Inconsistent findings in HME research

It is important to note that the literature on the relation between the HME and mathematical learning has produced inconsistent results, this is in stark contrast to the literature on the home literacy environment and its relation to reading outcomes (Morrison, Bachman & Connor, 2005). The majority of HNE studies used questionnaire based self-report measures of the frequency of home numeracy activities. Many studies have established a positive, unique impact of the frequency of HNE activities on mathematical development (Dearing et al., 2012; Kleemans, Peeters, Segers & Verhoeven, 2012; Manolitsis et al., 2013; Niklas & Schneider, 2014). In contrast, some studies have found no relation between HNE and a range of mathematical skills (e.g. Blevins-Knabe, Austin, Musun, Eddy & Jones, 2000; Missall, Hojnoski, Caskie & Repasky, 2015). Typically, research indicates that socio-economic status (SES) is related to mathematical development, (Galobardes, Shaw, Lawlor, Lynch & Smith, 2006; Mercy & Steelman, 1982; Sammons et al., 2004). Even when a relationship has been established, some studies have identified that after controlling for SES and parental attitudes the relationship does not persist (DeFlorio & Beliakoff, 2015).

Child characteristics

A recent review of published papers indicated that inconsistent findings may be attributable to differences in age of children within samples (Thompson, Napoli & Purpura, 2017). This narrative review indicated that HNE did not impact on mathematical outcomes of younger children (approximately 3-4-year-olds) but did moderately affect older children (approximately 5-6-year-olds). Although not specifically highlighted by the review, it is striking to note that across the thirteen included studies a wide variety of questionnaire measures were administered, with some overlapping content.

Psychometric properties

Some researchers who have created HNE scales have not provided adequate information about item generation and refinement, scale dimensionality, scale score reliability, or validity (e.g. Kleemans et al., 2012; LeFevre et al., 2009; Melhuish et al., 2008). As few questionnaires have been developed following best practice for scale development or have been validated beyond construct validity (e.g. LeFevre et al., 2009) the inconsistent results are perhaps unsurprising.

Characteristics of content and activities

There are many concepts that are captured in mathematics (i.e. numeracy, spatial skills, geometry, patterning) that are not captured in every frequency of activities questionnaire in the same way. For example, geometry is covered in both Hart et al. (2016; age 3-8 years old) and Missall, Hojnoski, and Moreano (2017; age 3-5 years old) questionnaires through different questions; “Fold or cut paper to make 3D objects”, “Play with legos” (Hart et al., 2016) and “Identify shapes in the everyday settings and activities”, “Put shapes together to make a larger shape” (Missall et al., 2017). The wide variety of skills that are encompassed by the concept “mathematics” and the variety of ways by which these skills can be measured could be a source of inconsistence in HME literature.

Some researchers have made distinctions between different types of activities using terms interchangeably, such as indirect versus direct and informal versus formal skills, with different definitions between studies (e.g., Anderson, 1998; LeFevre et al., 2009; Skwarchuk et al., 2014). LeFevre et al. (2009) conceptualised activities as either indirect or direct. Indirect activities were defined as those that are naturally occurring tasks that communicate mathematical information incidentally, for example playing boards games with dice, setting the table or weighing while baking. Direct activities are those that are used to directly teach mathematical skills or concepts to develop child’s mathematical skills, for example practicing simple sums and learning to identify number symbols. Skwarchuk et al. (2014) suggested that participating in formal practices would support the development of symbolic mathematics knowledge, while informal mathematics exposure would promote non-symbolic mathematics skills. Skwarchuk et al. (2014) found that formal home numeracy practices accounted for unique variance in children's symbolic number knowledge whereas informal exposure to games with numerical content predicted children's non-symbolic arithmetic performance, thus supporting their hypothesis. However, this hypothesised conceptual model of the HNE (Skwarchuk et al., 2014) has rarely been replicated. For example, there appears to be a differential effect of formal and informal activities on mathematical learning, with formal activities being positively related to attainment and informal activities being negatively related (Huntsinger, Jose, & Luo, 2016). Further, Huntsinger et al. (2016) found that participating in formal mathematics activities predicted both formal (learned through explicit instruction using rules, principles, and procedures e.g. calculations both addition and subtraction) and informal (acquired outside of formal schooling e.g. concepts of relative magnitude) mathematics knowledge, whereas engaging in informal activities predicted neither. Hence, dichotomisation of home mathematics activities does not seem to reduce the inconsistences in the literature.

In addition, some studies make a distinction between basic and advanced activities (Skwarchuk, 2009). Of course, the type of content of these two types of activities varies with age, for pre-schoolers advanced activities may include multiplicative counting, whereas this may be a basic task for a child in the early primary years. These developmental changes in children's skills have perhaps led to inconsistent findings on the relationship between HNE and mathematics skills. For older children, heightened frequency of advanced activities is associated with higher level mathematical skills, and the reverse for the frequency of basic activities (Skwarchuk, 2009; Skwarchuk et al., 2014). In contrast for younger pre-schoolers (i.e. 3-years old) the reverse is true, with more basic activities, rather than advanced, associated with higher attainment (Thompson, Napoli & Purpura, 2017). Of course, these four factors can overlap, but it is important that any validated questionnaire can record and assess this breadth of home-based activities for the targeted age group. In the context of conflicting results from a growing body of studies there is a clear need to develop and validate a coherent and inclusive measure of HME which is both reliable and valid.

Other considerations

In addition, it is uncertain that the items within currently published HME literature reflect the rapidly changing home environment of children (albeit mainly growing up in the Global West), specifically in relation to technology (OfCom, 2013, 2016). OfCom (2016) state that there are two devices in the home that continue to be used by children: television sets (92% for 3-4-

year olds and 96% for 5-7s) and tablets (55% for 3-4s and 67% for 5-7s). Thus, technology advances have potentially expanded the reach of maths learning in the home. Yet, questions about educational technology are rarely used beyond one question in HME questionnaire measures (e.g., How often did you and your child engage in the following activities? "Uses maths software" (Huntsinger et al., 2016) and "Playing counting games using child computer or arithmetic software" (Kleemans et al., 2012) and so on (e.g., Deflorio & Beliakoff, 2015; Skwarchuk & LeFevre, 2015). This makes it difficult to measure the extent of educational technology being used in the home and whether it makes a difference. Hence, this study aims to develop a measure that includes a variety of items regarding educational technology in relation to maths learning. This study will explore what types of items need to be included in a HME measure that reflect educational technology practices in the home environment. Through qualitative research Cahoon, Cassidy and Simms (2017) identified that parents regularly use technology with their preschoolers to support mathematical learning. Thus, failure to include multiple questions related to technology use in HME questionnaires may lead to misrepresentation of the home environment.

The HNE has sometimes been approached as a unidimensional construct (e.g. Blevins-Knabe & Musun-Miller, 1996; Kleemans et al., 2012) wherein all activities occurring in the home environment related to numeracy have been measured. Thus, many studies focus on the number activities and ignore other important areas such as technology and sibling interaction (Cahoon et al., 2017). Some HNE questionnaires do cover other mathematical domains such as geometry and shape (e.g. LeFevre et al., 2009). However, most HNE questionnaires use narrow terminology by using the term *numeracy*. It is necessary to be more consistent in communicating that both children's numerical skills and a broader range of mathematics skills (i.e., numeracy, spatial skills, geometry, patterning etc.) are being examined for their effect on the home environment (Belvins-Knabe, 2016; Hart et al., 2016), thus the term home *mathematics* environment may be more appropriate. In this paper the terminology HME has been used when broader range of mathematics skills are discussed (including numeracy, spatial skills, geometry, patterning).

Current study

There are many possible reasons for the inconsistent findings among HNE research; (1) the characteristics of the children participating to the studies, (2) the psychometric properties of the questionnaires that were used in previous studies and (3) the characteristics of the content and activities that the parents offer to these children. This study aims to systematically develop a comprehensive HME questionnaire that reliably assesses the experiences of pre-school children (3-5-year-olds), this new measurement tool will be referred to as the Pre-school Home Mathematics Questionnaire (PHMQ) as it involves home environment relevant dimensions beyond numeracy. The questionnaire was developed using the framework of Learning Trajectories (Clements & Samara, 2004) reflecting the learning goals and activities that children might engage in (Simon, 1995). Most HME questionnaires have been developed and used in home environments that reflect the developed world, for example Canada. This is the first study within the UK that has created an HME questionnaire that is culturally specific, were items are not just deductive and drawn from other HME questionnaires such as Melhuish et al. (2008). Hence, the aim of the PHMQ is to develop a culturally appropriate HME questionnaire that shows good psychometric qualities for 3-5-year olds growing up in the UK. This specific age-related focus is important due to the varied nature of activities that are appropriate across development. It is of utmost importance that this new measurement instrument demonstrates strong psychometric properties (i.e. reliability and validity, Hinkin, 1998; Schoenfeldt, 1984). The creation of measurement tools should ideally be both inductive and deductive (Williamson et al., 1982), an approach unique to this current study of scale development. An advantage of using both deductive and inductive approaches to scale development is that it increases the chances of content validity in the final scale (Hinkin, 1998).

To develop and validate the dimensions of the PHMQ and produce an instrument with evidence of reliability and validity, this study has followed recent scale development and validation research processes (e.g. Hinkin,1998; Nunes, Pretzlik & Ilicak, 2005). Overall, four studies are included in this paper that support the examination of construct, content, and criterion validity. The ultimate objective of this scale development and validation process is to ensure that the new PHMQ measure aligns with APA standards for psychometric adequacy (APA, 1995; Hinkin, 1998)

Method

Table 1 provides an overview of the processes involved in each of the four studies within this paper that ensure rigorous development and validation methods of the PHMQ. As there are four studies involved in this current paper, each study begins with an overview followed by the method and results of the study. The only study that does not follow this structure is study one which involves an overview and method only. The reason for this is because this study involves generating items based on previous interview transcripts (Cahoon et al., 2017) and it is believed all relevant information is provided for the reader to understand how the items were generated, including information provided in the supplementary information of this paper (See Appendix 2, Table 1).

Each study in this paper was reviewed and approved by School of Psychology Research Ethics Committee before the study commenced. Signed written consent was obtained from all participants. For each study the criteria for participation was that the parent/guardian defined themselves as the primary carer for a children of pre-school age.

Table 1. PHMQ Development and Validation Procedures

Studies	Step by step	Details of studies
Study 1: Item generation	The creation of items that assess the construct of the home mathematics environment. Scale design and coding of responses (e.g. Likert scales). Initial item generation; 44 inductive and 25 deductive items developed. 8 home environment dimensions within the Pre-school Home Mathematics Questionnaire (PHMQ): 1) Parent expectations, 2) Child maths literacy, 3) Counting ability, 4) Parent-child teaching methods, 5) Target child-sibling interactions, 6) Frequency of maths activities scale, 7) Child's understanding of numeracy, and 8) Support question.	Parent interviews (N = 8). Parents had children aged 37 months to 59 months, M_{age} = 47.5 months. Content analysis.
Study 2: Questionnaire refinement	Confirm duration and presentation of questionnaire in pre-pilot. Questionnaire administration to parents/guardians. Exploratory factor analysis (EFA) of the frequency of maths activities scale. Questionnaire refinement (e.g. due to lack of variability in responses).	Student population pre-pilot (N = 10). Questionnaire administration (N = 172). Parents/guardians had children aged 36 months to 60 months, M_{age} = 46.2 months. Exploratory factor analysis. Questionnaire refinement. Student population re-test (N = 10).
Study 3: Scale validation (qualitative)	Pilot PHMQ to confirm refinement of questionnaire with parents/guardians. Parent interviews for content and criterion validity. Content validity – considers whether appropriate questions have been asked in the PHMQ. Criterion validity – investigates contrast cases of parents with very high or very low scores on each of the themes within a questionnaire and compares the contrasting cases to the interview responses.	Pilot PHMQ with parents/guardians to confirm refinement (N = 30). Parent interviews (N = 8). Parents/guardians had children 36 months to 49 months, M_{age} = 42.8 months. Content validity. Criterion validity.
Study 4: Scale validation (quantitative)	Confirmatory factor analysis (CFA) of the frequency of maths activities scale. Construct validity. Reliability. Factor loadings.	Questionnaire administration (N = 136). Parents/guardians had children 43 months to 54 months, M_{age} = 48 months. Confirmatory factor analysis.

Study 1: An overview

Following construct definition, item generation (Table 1: Study 1) featured individual, in-depth interviews with eight parents with pre-school aged children (i.e. 37 months to 59 months, M_{age} = 47.5 months), using the same transcripts used in previous literature (Cahoon et al., 2017). The interviews were exploratory and aimed to gain opinions from parents on their everyday routine activities and understand the way in which parents encourage the development of early numeracy skills in the home. The six themes that were identified from the thematic analysis were; (1) numeracy environment structure, (2) frequency of number-related experiences, (3) levels of number knowledge, (4) views of technology, (5) parent-child interactions and (6) social interaction. The diversity of the themes illustrated how the HME may be influenced by parents' views and experiences of numeracy-related activities and children's interactions with others. For instance, (1) the numeracy environment structure theme demonstrated the types of environments that parents create for their children to learn numeracy in the home. Initially participants state that teaching mathematics should be instinctive but admitted that it is difficult to spontaneously formulate plans. Findings showed that parents may not always be cognisant when undertaking numerical activities with their child in the home and hence the HME is largely unstructured (see Cahoon et al., 2017 for more detail). Through the thematic analysis used within this paper (i.e. Cahoon et al., 2017) the theoretical foundation for the PHMQ was developed. These transcripts were then used to generate items for the PHMQ using content analysis. In addition, previous questions from other questionnaires were identified and incorporated into the PHMQ.

Method

Item generation

Using content analysis, this inductive approach developed 44 items to create the initial PHMQ. Further, the deductive item generation method developed a base set of items that assessed the HME drawn from previous HME measures (e.g., LeFevre et al., 2009; Lukie et al., 2014; Kleemans et al., 2012; Melhuish et al., 2008) and previous parent-child interaction research, such as observational research involving parent guidance and support (e.g. Bjorklund, Hubertz & Reubens, 2004; Vandermaas-Peeler, Boomgarden, Finn & Pittard, 2012). All items were cross-referenced between those mentioned from the interviews (e.g. a numeracy activity such as counting objects) and items from other HME measures or cited in previous parent-child interaction research. Together, the deductive items (N = 25) were combined with the inductive items (N = 44) and therefore totalled to 69 items. Thirty-eight items focused on the frequency of maths activities. Additional questions investigated more nuanced factors, such as interaction with parents and siblings. It is acknowledged that there are more numeracy items than maths items at this stage. However, this would be reflective of the age group that the PHMQ is targeted towards (i.e. ages 3-5 years). Therefore, the activities are developmentally appropriate (see supplementary information, Appendix 2, Table 1 for a detailed breakdown of the items, how each item was generated and initial item reduction criteria).

Study 2: An overview

Questionnaire refinement (Table 1: Study 2) involved parents with children aged 3 to 5 years old taking part in the PHMQ. The aim of Study 2 was to examine how well items confirmed expectations concerning the psychometric properties of the new measure (Hinkin, 1998), including examining whether items produce necessary variance for subsequent statistical analyses (Stone, 1978). Study 2 addressed construct validity, which incorporated two psychometric properties, factor structure and scale score reliability. Furthermore, it is important that the response scale used (e.g. rank order or rating items) for the items produces necessary variance for subsequent statistical analyses (Stone, 1978).

Method

Participants

A total of 172 parents/guardians took part in the PHMQ. To acquire an equal spread of participants across SES through data collection, the proportion of free school meals (FSM) per school was calculated across Northern Ireland, using Department of Education (2014) statistics. It should be noted that FSM is an imperfect proxy of mothers and partners' education and social class (Hobbs & Vignoles, 2007). Therefore, to avoid imperfect proxy bias (i.e. a proxy that correlates with the key variable but cannot be understood in isolation) parents were asked in the PHMQ to complete 8 questions from the National Statistics Socio-economic Classification (NS-SEC; Rose & Pevalin, 2010), which allowed the researcher to derive SES using the Standard Occupational Classification (SOC2010). The FSM statistics were divided into three proportions to distinguish schools that had low (4-18%), medium (19-58%) and high (59-85%) FSM eligibility. The average FSM eligibility was 37.7%. It was anticipated that an equal spread of pre-schools would be contacted from the three FSM eligibility categories. However, there was a low participation rate from the pre-schools in the medium FSM eligibility category, so more pre-schools were contacted from this category. Participants were recruited through 11 local pre-schools and two privately owned soft-play centres. A soft-play centre is a soft obstacle play area for children up to the age of 8 years-old at which parents/guardians supervise play. Thus, it was deemed an ideal area to target parents with children aged between 3 to 5 years old. The proportion of PHMQ returned from each of the low, medium and high FSM categories were 30%, 42.5% and 27.5%, respectively.

The sample consisted of 148 mothers, 18 fathers, 3 grandparents, 2 foster parents and 1 adoptive parent. The target child that parents/guardians were responding to questions about were aged between 36 months to 60 months (M_{age} = 46.2 months; 52.3% female), with 85.5% of the target pre-school child having sibling/s (N = 147). The parents/guardians were between 23 and 65 years old (M_{age} = 35.26 years). SES data was converted into a three-class categorical variable as described in NS-SEC (2010), this can be assumed to involve a form of hierarchy: *high SES* (50.7%), *middle SES* (17.5%), and *low SES* (25.5%).

Procedure

The questionnaire was piloted (N = 10) to assess completion time and the ensure that the presentation was easy to read and understand. The questionnaire took approximately 10-15 minutes to complete and adjustments were made to the questionnaire to make sure participants would understand the terminology. After these changes were made the PHMQ was tested. No pilot data, at any point in this study, was used in analysis (e.g. for the exploratory factor analysis (EFA)). The participants that completed the PHMQ in the play centres did the questionnaire on the day they agreed to the study and they did not take them home. The participants who completed the PHMQ in the pre-schools returned the PHMQ to the child's teacher for collection in sealed envelopes to maintain confidentiality.

Data analysis

Data was entered by two researchers and was verified to ensure 100% validity. A subject-to-variable ratio of 1:4.5 was achieved with 172 participants and 38 variables included in the EFA. This is consistent with previous research which suggests that a ratio of 1:3-1:6 subject-to-variable is acceptable (Arrindell & Van der Ende, 1985; Cattell, 1978).

Results

Questionnaire refinement

The PHMQ consisted of eight dimensions: (1) *parent expectation* of their children's academic success, (2) child *maths literacy*, (3) child *counting ability*, (4) *parent-child teaching methods* (e.g. what are the specific things parents say or do to encourage and support their child to learn maths?), (5) *target child-sibling interactions* (e.g. what numerical activities siblings are most likely to do together?), (6) parent's view of their *child's understanding of numeracy*, (7) caregivers *support of numeracy learning* in the home and (8) *frequency of maths activities scale.* See supplementary information Appendix 2, Table 1 for a detailed breakdown of the items, how each item was generated and initial item reduction criteria.

The first three dimensions, mentioned above, are known as benchmark questions as they give context to results by allowing comparison between participant responses. These are essential questions that gauge the background of the parents expectations for their child and the child's ability level. Each of these three dimensions had good variance and were retained for the final PHMQ. The next two dimensions *parent-child teaching methods* and *target child-sibling interactions* were named as interaction questions as they involve the target child interacting with both parents and siblings. The section *parent-child teaching methods* was kept due to good variation in results. However, the *target child-sibling interactions* (originally 13 questions) were reduced due to lack of variability, potentially explained by "halo effect" (i.e. parents wanting their child to be perceived favourably by reporting that they take part in an activity that may be too advanced for them). This finding was also discovered in the previous qualitative interviews (Cahoon et al., 2017). Therefore, 11 ranking options for *target child-sibling interactions* were reduced to 7 ranking options. The threshold for cut off was any rank option that scored over 20% in the least likely categories. The reason for reducing rank order options was that participants found it too difficult to rank order 11 options. However, after reduction to 7 ranking options this question piloted (N = 10) again and was still found to be difficult to complete. Therefore, this question was changed to match the 5-point Likert scale of the frequency of maths activities scale. The parent's view of their *child's understanding of numeracy* and caregivers *support of numeracy learning* in the home were removed due to lack of variability in results. There was a lack of variance in these questions indicating that they were classic "halo effect" questions (Fitzpatrick, 1991; Wilson, Hewitt, Matthews, Richards & Shepperd, 2006).

The *frequency of maths activities scale* of 38 questions, were analysed using EFA to investigate variable relations for this complex concept. These items were analysed using a principle components analysis with oblique rotation (direct oblimin). Table 2 summarises the factor loadings after rotation for the *frequency of maths activities scale*. The Kaiser-Meyer-Olkin measure verified the sampling adequacy for the analysis (KMO = .80), and all KMO values for individual items were greater than .59. Five factors, comprising 28 items, had eigenvalues over Kaiser's criterion of 1 and in combination explained 53.14% of the common variance. The factors were labelled as follows; (1) parent - child interactions, (2) computer maths games, (3) TV programmes, (4) shape and (5) counting. Ten items did not load onto any factor and therefore these were removed from further analysis and questionnaire administration. To note, there is an item that factored into the parent - child interactions subscale that involves shape (i.e. Asking shape related questions (e.g. "how many sides does a circle have?")). However, theoretically this makes sense as this activity would involve a parent interacting with their child to ask shape related questions. Cronbach's alpha for the full scale was .89. Cronbach's alpha for the subscales were acceptable, ranging from .76 for the *counting* factor to .81 for both the *parent - child interactions* and *computer maths games* factors, thus display good internal reliability.

Overall, from the initial 69 items, 19 items (14 deductive and 5 inductive generated items) were removed for different reasons mentioned previously. Thus, a total of 50 items were retained. Through the item reduction process, the PHMQ contained six home environment dimensions; (1) *parent expectation* of their children's academic success, (2) child *maths literacy*, (3) child *counting ability*, (4) *parent-child teaching methods*, (5) *target child-sibling interactions* and (6) *frequency of maths activities scale*. The dimensions called child's understanding of numeracy and support questions were removed.

Table 2. Summary of Exploratory Factor Analysis Results for Frequency of Maths Activities Scale

Items	Rotated Factor Loadings				
	Parent – child interactions	Computer maths games	TV programmes	Shape	Counting
Identifying names of written numbers	.65	-.01	.11	.08	.03
Write numbers	.59	.05	.06	.03	.14
Teaching about measurements (e.g. baking or height)	.54	-.04	-.03	.02	-.18
Time terminology (e.g. big hand, little hand)	.50	.09	-.05	-.03	-.07
Asking shape related questions (e.g. “how many sides does a circle have?”)	.49	.07	-.09	.20	-.12
Scenarios number games (e.g. "if I have two toy cars and I take one away, how many cars do I have?")	.49	.10	.04	-.09	-.25
Teaching about money (e.g. playing shop or buying sweeties)	.43	.12	-.10	.02	-.28
Sticker books	.38	-.02	.18	.14	-.08
Maths related websites (e.g. coolmaths.com)	-.02	.71	.00	-.05	.01
Racing games (e.g. the faster they complete sums, the faster the boat moves)	-.17	.67	.03	.01	-.02
Size/matching apps (e.g. “put the big skirt on the small girl”)	-.03	.65	.04	.07	-.01
Maths applications (e.g. Number Jacks)	.19	.63	.10	-.05	-.01
Add and subtraction games	.20	.60	.09	-.04	.01
Filling in the gap number games (e.g. what is next in the sequence?)	.16	.51	-.06	.07	-.01
Watching number related TV shows	.13	.00	.89	-.07	-.03
Rhyming TV shows involving numbers (e.g. Number Jacks)	.03	.11	.85	.02	-.04
Watch educational programs (e.g. Dora the Explorer)	-.15	.13	.38	.14	-.19

Sorting shapes	-.03	.06	-.03	.62	-.19
Sorting objects by size	-.05	.04	-.04	.61	-.34
Creating patterns with objects (e.g. arranging blocks into shapes)	.10	.12	-.12	.61	.02
Playing with building blocks	-.04	-.04	.12	.58	.15
Play with jigsaws	.09	-.12	.08	.54	.04
Pairing/matching games	.07	.09	-.03	.44	-.13
Counting out food, dinner plates, knives and forks	-.04	.01	.11	.09	-.61
Counting	.07	-.09	.05	-.07	-.59
Counting objects (e.g. ducks in bath, blocks, new toys, books)	.04	.05	.06	.15	-.55
Counting on fingers/hands	.15	.01	.02	.01	-.55
Comparing sets of objects (e.g. brother has more than mum)	.09	.20	-.03	.05	-.52
Eigenvalues	7.16	2.39	2.14	1.67	1.53
% of variance	25.58	8.52	7.63	5.95	5.45
a	.81	.81	.79	.78	.76

Note: Factor loadings over .40 appear in bold. N=172

Study 3: An overview

After the development of the PHMQ, the scale validation process involved two studies, the first being qualitative to assess content and criterion validity (Table 1: Study 3). Content validity considers whether appropriate questions have been asked in the measure (Nunes et al., 2005). It allows for comparison of the themes identified in the questionnaire with those emerging in subsequent interviews (Nunes et al., 2005). Criterion validity investigates contrast cases of participants with very high or very low scores on each of the themes within a questionnaire and compares the contrasting cases to the interview responses (Nunes et al., 2005). This enhances the validity of the dimensions included within the PHMQ.

Method

New dimension

At this stage before the following semi-structured interviews were conducted, a new dimension called the number game checklist was developed and added to the PHMQ. Skwarchuk et al. (2014) created a measure to assess the informal numeracy experiences by developing a number games title checklist. This framework was adapted from Sénéchal and LeFevre (2002) study that used parent's knowledge of storybook titles as a proxy measure of informal home literacy practices. The number games title checklist by Skwarchuk et al. (2014) was created for a Canadian sample, thus a new, culturally appropriate, number games checklist was developed as a measure of *informal home numeracy practices* (number games exposure checklist) so that the games were relevant to the United Kingdom (UK). The rationale for the inclusion of this dimension is so that the measure can assess *informal home numeracy practices* through parent's recognition of board games alongside the frequency of maths activities scale that potentially assesses both formal and informal home numeracy practices. Note that a two-factor model based on the original definitions of direct and indirect numeracy activities by LeFevre et al. (2009) was explored through a confirmatory factor analysis in the supplementary information (See Appendix 5 for details).

To develop the board game checklist information was gathered about commercially available board games suitable for children aged 3 to 6 years both in store and online from three retail establishments. To compile the list of games, selection criteria were used to allow parents a chance to have knowledge of the games. In Sénéchal, LeFevre, Hudson, and Lawson (1996) *book title checklist* fairy tale games (i.e. those games that involved fairy tale characters from movies or television) for which a movie or television version existed were eliminated due to possible over familiarisation. To allow for the games to be readily available to parents only those game titles that were available in two of the three retail establishments were selected.

Lastly to ensure that the games were accessible to all parents regardless of income level only games that were under £15 were selected. Games were categorised according to whether they included numerical components (counting, adding and recognising numbers). In contrast to Skwarchuk et al. (2014) that included 25 titles (10 numerical games, 10 non-numerical games, and 5 plausible but non-existent games), this board game checklist consisted of 30 game titles; 10 numerical; 10 non-numerical and 10 plausible but non-existing games. The number of plausible but non-existent games was increased to 10 as this was equal to that of the numerical and non-numerical game.

The newly created number games checklist was cross-referenced with Skwarchuk et al. (2014) number game exposure checklist. Four numerical, 2 non-numerical and 1 plausible but non-existing games were taken from Skwarchuk et al. (2014) checklist as they also reached the selection criteria used in this study. As in previous home numeracy research (Skwarchuk et al., 2014), parents were asked to indicate their familiarity with children's game titles. Parents were asked not to guess or stop to verify any game titles online or in a catalogue. Participants were informed that non-existing games were included in the checklist to minimise guessing. To calculate the number game checklist score, the total of correctly marked number games was corrected for guessing (e.g. if 7 number games and 1 non-existing games were selected, this was scored as (7-1/10) x 100 = 60%; Skwarchuk et al., 2014).

Therefore, overall the PHMQ was made up of seven-home environment relevant dimensions including the *informal home numeracy practices* (number game exposure) section to the PHMQ. The updated PHMQ with the seven-home environment relevant dimensions was subsequently piloted with parents/guardians to confirm refinement (N=30).

Participants

Eight participants (M_{age} = 37.8 years) agreed to take part in the PHMQ and the interview; 6 mothers, 1 father and 1 grandparent. The target child (50% female) were aged between 36 months to 49 months (M_{age} = 42.8 months). Data saturation was reached in the eight interviews which is consistent with other studies (Isman, Ekéus & Berggren, 2013; Isman, Mahmoud Warsame, Johansson, Fried & Berggren, 2013). Data saturation was achieved when further coding was not achievable, thus the ability to obtain additional new information from further interviews was no longer possible (Fusch & Ness, 2015).

Procedure

The interviews took place at two soft-play centres that had been used as sites in Study 2. The topic guide of questions asked to the parents included questions such as, "Do you think your child is interested in maths? If so, why?" and "Can you compare the frequency and structure of mathematical activities to reading at home?". These questions were used as they were deemed appropriate to gain sufficient information for content and criterion validity as these were the same questions asked in the original interviews (Study 1; Cahoon et al., 2017). Half of the participants were administered the questionnaire before the interview and half of the parents were given the questionnaire after the interview. The individual interviews lasted approximately 40 minutes and the PHMQ took approximately 10 minutes to complete. The interviews were recorded and transcribed before analysing.

Data analysis

The subscales in the frequency of maths activities scale were used to assess content and criterion validity. The other dimensions from the PHMQ such as the frequency of reading compared to numeracy, target child-sibling interaction, structure of the home numeracy environment and parent-child teaching methods, will be evaluated to assess the content validity of the PHMQ. The parents 'responses were coded using NVivo (Version 11) into content categories based on the five subscales within the frequency of maths activities scale. Criterion validity was assessed through contrasting cases that were identified by obtaining the total scores for the five subscales and were calculated for each participant. Scores ranged from 0 to 4, based on a 5-point Likert scale. Respondents with low scores were more likely to answer that an activity did not occur and hence would have a score closer to 0. Respondents with high scores would be more likely to answer that an activity occurred almost daily and thus score closer to 4. The parents 'interview transcripts were then searched for comments relevant to the subscales.

Results

Content validity

There was an agreement between parents 'views in the interview and those assessed by the PHMQ. Issues surrounding the six dimensions of the PHMQ (i.e. (1) *parent expectation* of their children's academic success, (2) child *maths literacy*, (3) child *counting ability*, (4) *parent-child teaching methods*, (5) *target child-sibling interactions* and (6) *frequency of maths activities scale*) and the five frequency of maths activities subscales were mentioned in the eight interviews and used to assess the content validity. The definitions and sample comments illustrating each dimension and factor are summarised in supplementary information (See

Appendix 3, Table 2). Appendix 3, Table 2 shows that each factor was mentioned from the *frequency of maths activities* scale in the interview thus, all items were retained in the scale.

Emerging topic

An emerging topic was identified in the interviews, which suggested the need for increasing the breadth of the *frequency of maths activities* scale. YouTube was mentioned by half the participants. Children watched a range of videos including educational videos. Some examples of the types of videos children watched are reported below;

> *"She likes watching a couple of YouTube videos. She loves the videos where people open, they are called blind bags, the likes of My Little Pony or Paw Patrol. It's almost like a kinder eggs surprise thing and it will have one of the characters in them. She counts the characters sometimes" – Participant 3*

> *"Oh Number Jacks. He has only started to watch Number Jacks on YouTube. He likes that. It's quite good" – Participant 4*

> *"They usually watch YouTube videos. People have made up YouTube videos using the characters from Frozen or Paw Patrol or whatever and changing their colours or do the finger family" – Participant 6*

Summary of Content Validity

Two new additional topics arose from the current interviews that were not mentioned in the previous interviews. The first was that parent's interest in mathematics may influence the frequency of maths activities occurring in the home, therefore a question related to this topic was added to the PHMQ. The second was that parents reported watching videos with mathematics content on YouTube with their children, therefore the item 'Maths related YouTube videos 'was added to the frequency of maths activities scale. Overall, the analysis of interviews confirmed the dimensions included in the PHMQ.

Criterion validity

It is only possible to use criterion validity on the frequency of maths activities scale within the PHMQ as criterion validity involves contrasting cases of high and low scores, possible through a Likert scale. Analysing contrasting cases indicates that in the frequency of maths activities subscales there are differences between the extreme high and low scores. The high and low contrasting cases with sample comments illustrating each subscale dimension are summarised in supplementary information (see Appendix 4, Table 3). Some noteworthy findings were that time limits were important with regards to the frequency of computer usage; this is one reason for the varying frequencies of *computer maths games* subscale. The types of TV programmes being watched may influence the frequency and perhaps be one reason for the contrasting cases. This would be expected as the *TV programmes* subscale only involves questions about educational programmes. Therefore, those children who are mostly watching non-educational TV programmes would score low on the *TV programmes* subscale. It is important to note that a child's interest plays a factor in the TV programmes they want to watch (Cahoon et al., 2017) and this could influence high and low frequencies on this subscale. Nevertheless, the subscale seems to identify contrasting cases well. Parents who scored the lowest on the *counting* subscale stated that counting was mostly brought up by the parent. Whereas parents who scored high on the *counting* stated that mathematics was brought up naturally by their child and hence counting may be covered more often in the home if both the parent and the child are likely to bring up counting. Overall, there are clear differences between the views of parents with contrasting frequency scores as assessed through the interviews.

Study 4: An overview

As previously stated, the scale validation process involved two studies the second being quantitative; construct validity (i.e. confirmatory factor analysis; Table 1: Study 4).

Method

Participants

152 parents with children aged 43 months to 54 months (M_{age} = 48 months) agreed to complete the PHMQ. 136 (89%) participants (91% female) returned the PHMQ and were 34.9 years-old (SD = 5.7, Range 21-46 years). The same FSM classification approach was used as study 2. The proportion of PHMQ returned from each of the FSM Eligibility categories were 32%, 50% and 18%, respectively. There were 39.5% parents from *high SES*, 19.7% from *middle SES* and 23.7% from *low SES* backgrounds. The additional 17.1% represents missing data or that the responding parent had never worked, had been long-term unemployed or was a full-time student.

Procedure

The parent of the target child was asked to complete the PHMQ. Parents who complete and return the PHMQ were entered into a prize draw for a £50.00 Amazon voucher. The PHMQ was returned to the pre-school teacher and collected by the researcher.

Data analysis

A confirmatory factor analysis (CFA) was all completed in Mplus Version 1.5 (Muthén & Muthén, 1998-2017). Mplus was used to examine the factor structure instead of SPSS as Mplus allows for the researcher to place each item in the factor suggested by the exploratory factor analysis to test if the model fits. A CFA was utilised on the five subscales found in Study 2.

Results

Construct validity

A CFA with robust maximum likelihood was conducted in Mplus. This approach has been widely used in CFA models when continuous observed variables slightly or moderately deviate from the normality and it is superior to maximum likelihood (Li, 2016). In Figure 1 the five-factor model is presented.

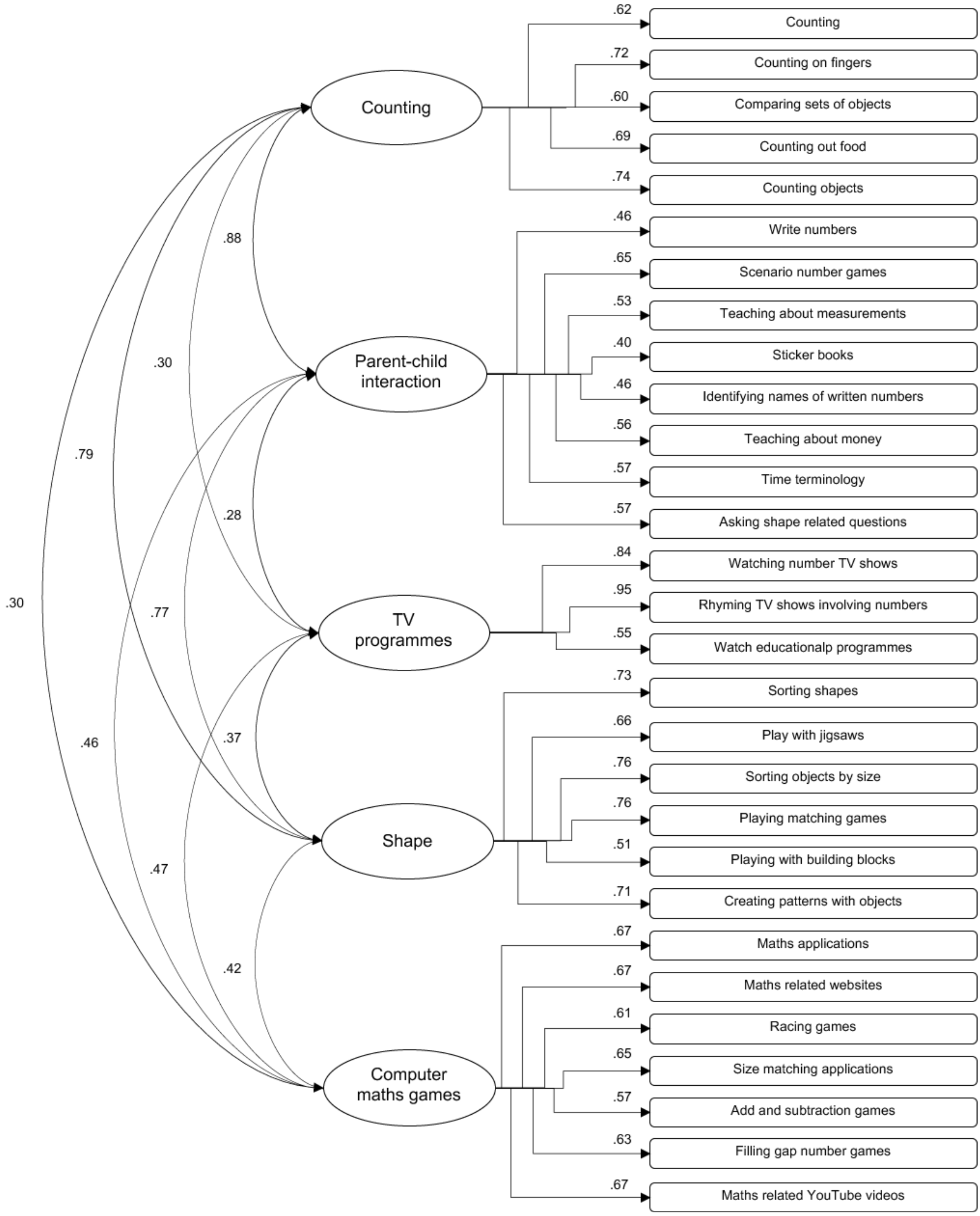

Figure 1. Five-Factor Model of the Frequency of Maths Activities Scale in the PHMQ

The selection of the most appropriate model was based upon goodness of fit statistics (Table 3). For more information on other models that were examined (i.e. One factor, total frequency of maths activities, five-factor second order models and two-factor model based on the original definitions of direct and indirect numeracy activities by LeFevre et al. (2009)) refer to the supplementary information (See Appendix 5). The model had acceptable model fit indices reporting a Comparative Fit Index (CFI) of 0.83 and a Tucker Lewis Index (TLI) of 0.81. Good fitting models are indicated by a CFI of > 0.95 (better model: > 0.97) and the same cut-off value for TLI applies (Geiser, 2012). A CFI > 0.90 is often regarded as an indicator of an adequate model fit (Awang, 2012; Coroiu et al., 2018; Hair, Black, Babin & Anderson, 2010) the same cut-off value for TLI applies (Awang, 2012; Coroiu et al., 2018; Forza & Filippini, 1998).

The CFI and the TLI are incremental fit indices that compare the fit of the target model to the fit of a baseline model (Geiser, 2012). In Mplus the baseline model, also known as the null independence model, assumes that the population covariance matrix of the observed variables is a diagonal matrix, in other words, it is assumed that there is no relation between any of the variables (Geiser, 2012). As a consequence, it is possible that the null model is "too good", meaning that the average level of correlations in the current data is rather low. In this case, Kenny (2015) argued that CFI should not be computed if the RMSEA (i.e. Root-Mean-Square Error of Approximation) of the null model is less than 0.158 as the CFI obtained will be too small a value (Beldhuis, 2012; Kenny & McCoach, 2003). When investigating the RMSEA values the model demonstrated acceptable RMSEA values (< 0.08) (Awang, 2012), the RMSEA value was 0.07. Therefore, the five-factor model is a reasonable model.

The SRMR (i.e. Standardised Root Mean Square Residual) coefficient is a standardised measure for the evaluation of the model residuals, however SRMR is somewhat biased by sample size. Marsh, Hau and Wen (2004) state that the SRMR values for solutions based on small sample sizes are unacceptable (greater than 0.08), whereas those based on large sample sizes are acceptable. A value < 0.08 is generally considered a good fit (Hu & Bentler, 1999). Therefore, taking into consideration all fit criteria for assessing goodness of fit the model presents acceptable fit indices (CFI = 0.83, TLI = 0.81, RMSEA = 0.07, SRMR = 0.072), thus it seems reasonable that a five-factor model be deemed a suitable measurement model.

Table 3. Model Fit Statistics for the Alternative Models of Frequency of Maths Activities

Model no.	Model explained	χ^2(p)	df	CFI	TLI	RMSEA (90% CI) p	SRMR	AIC	BIC	Sample-Size Adjusted BIC
1	One factor model	992.714 (0.00)	377	0.57	0.53	0.110 (0.101 – 0.118) 0.00	0.109	11841.537	12094.938	11819.717
2	Five-factor model	633.871 (0.00)	367	0.81	0.79	0.073 (0.063 – 0.083) 0.00	0.078	11480.253	11762.780	11455.925
3	Five-factor second order model	644.616 (0.00)	371	0.81	0.79	0.074 (0.064 – 0.083) 0.00	0.081	11482.686	11753.563	11459.361
4	Two-factor model (original direct and indirect activities)	992.703 (0.00)	376	0.57	0.53	0.110 (0.102 – 0.118) 0.00	0.109	11842.808	12099.121	11820.737

Note: Estimator = MLR; n = 136; $\chi 2$ **=** Chi-square Goodness of Fit statistic; *df* = degrees of freedom; p = Statistical significance; CFI = Comparative Fit Index; TLI = Tucker Lewis Index; RMSEA (90% CI) = Root-Mean-Square Error of Approximation with 90% confidence intervals; BIC = Bayesian Information Criterion; AIC = Akaike information criterion

The additional 'Maths related YouTube videos 'item.

As discussed in Study 3, an additional item was discovered through the process assessing content validity and added into the frequency of maths activities scale. This item was named 'Maths related YouTube videos'. As confirmed by the interviews with parents during content analysis, younger children mostly use YouTube to consume traditional, 'TV-like 'content (Ofcom, 2016). Therefore, the item 'Maths related YouTube videos 'was initially added to the *TV programmes* subscale of the frequency of maths activities scale.

However, on examination of the modification indices (i.e. restrictions that may be relaxed to obtain a significant improvement of the global model fit; Geiser, 2012) it was apparent that the item, 'Maths related YouTube videos', should be placed within the *computer maths games* subscale which made for better model fit indices. The fit indices for the new item placed in the *TV programmes* subscale were CFI = 0.81, TLI = 0.79, RMSEA = 0.073, SRMR = 0.078. Whereas, the fit indices for new item placed in the *computer maths games* subscale were CFI = 0.82, TLI = 0.81, RMSEA = 0.070, SRMR = 0.072. As suggested by the modification indices and the model fit statistics the new item was placed in the *computer maths games* subscale. This was the only suggested modification indices, further evidence that the five-factor model is a suitable measurement model.

Discussion

By following the procedures used by Hinkin (1998) and Nunes et al. (2005) the new PHMQ measure demonstrates construct, content, criterion validity and satisfies APA standards for psychometric adequacy (APA, 1995; Hinkin, 1998), which was the ultimate objective of this scale development and validation process. The scale development process (Table 1: Study 1 and 2), presented construct validity, which addressed two psychometric properties. Firstly, the five-factor structure of the frequency of maths activities scale found through the EFA demonstrated that the *factor structure* and *scale score reliability* had high levels of reliability (a = .76 to .81).

This high level of reliability is consistent with other studies in which a factor analysis was used to refine the home numeracy environment measure. For instance, LeFevre et al. (2009) reported a reliability between .71 and .84 for their numeracy-related activities measure comprising of four factors; (1) number skills, (2) games, (3) applications and (4) number books. Kleemans et al. (2012) established two factors in their home numeracy questionnaire, (1) parent-child numeracy activities and (2) parents 'numeracy expectations, with a reliability of .76 and .83, respectively. Further, Lukie et al. (2014) established a four-factor model, (1) exploratory cognitive play, (2) active play, (3) crafts, and (4) screen time, within their child-interest scale with a reliability ranging between .60 to .79. LeFevre et al. (2009) used factor analysis to classify activities reported in the (1) number skills and (4) number books subscales as *direct* teaching activities and the (2) games and (3) application factors as *indirect* experiences. However, the results of the factor analysis in the current study does not replicate LeFevre et al. (2009) findings of direct versus indirect experiences, instead five separate subscales were identified, (1) parent – child interactions, (2) computer maths games, (3) TV programmes, (4) shape and (5) counting.

Each of the studies mentioned above contribute to the growing body of research on the influence of the home environment on mathematical development. However, the unique aspect of the current PHMQ measure is its rigorous development through use of both deductive and inductive approaches. Skwarchuk (2009) drew numerical content from a questionnaire, diary entries and videotaped play sessions in a Canadian setting. Similar to Skwarchuk (2009) the aim was to draw out mathematical content that occurred in the home through interviews within a UK content. Literature demonstrates equivocal definitions (Cahoon et al., 2017); rendering is difficult to determine what defines an effective home numeracy environment that facilitates development in mathematics. This is further complicated by the lack of agreement on what parental involvement and interactions matter most. The current study broadens the definition of the HME through interviews with parents, allowing items to be generated inductively and therefore developing a comprehensive measure of the HME for preschool children following well-established procedures such as Hinkin (1998) and Nunes et al. (2005). This rigorous approach ensures that the measure captures the actual HNE that young children experience.

The scale validation process (Table 1: Study 3 and 4) consisted of content and criterion validity. Content validity demonstrates that the themes included in the PHMQ are raised by parents in the interviews. The examination of criterion validity showed that there were clear differences between the views and experiences of parents with low and high scores across all five PHMQ subscales. One of the new items that was spontaneously raised by the parents was that their children watched a range of videos on YouTube, including educational videos. YouTube is predominantly utilised, with 37% of 3 to 4-year-olds and 54% of 5 to 7-year-olds, using the YouTube app or website (Ofcom, 2016). As confirmed by the interviews with parents, younger children mostly use YouTube to consume traditional, 'TV-like' content (Ofcom, 2016). Therefore, the item 'Maths related YouTube videos 'was placed within the *computer maths games* subscale within the frequency of maths activities scale based on the model fit indices from the CFA. A CFA was used to quantitatively assess the quality of the five-factor structure of the frequency of maths activities scale offering evidence of the construct validity of the scale

(Hinkin, 1998). Taking into consideration all criteria for assessing goodness of fit the five-factor model it was deemed a suitable measurement model, confirming the findings from the EFA (Study 2).

Overall, there are more numeracy-based items than mathematics-based items within the PHMQ. This is reflective of the target age group (ages 3-5 years) for the Pre-school Home Mathematics Questionnaire. Therefore, the activities included in the questionnaire are developmentally appropriate. The questionnaire is titled the Preschool Home *Mathematics* Questionnaire due to broader items than simply numeracy being included, such as shape and patterns. Similar to Clements, Sarama & Liu (2008), who created a measure to access the mathematical knowledge and skills of children aged three to seven years, the PHMQ broadly covers mathematics and would be proportional for the amount of non-numeracy maths presented in preschool.

Contribution to research

As far as the authors are aware, this was the first study that uses both an inductive and deductive approach to develop an HME questionnaire, which increases the chance of content validity in the final scale (Hinkin, 1998). Previous scales (i.e. frequency of number activities scales) have rarely gone beyond creating items using a deductive approach. Further, these scales have rarely been validated beyond construct validity (e.g. LeFevre et al., 2009). Schoenfeldt (1984, p.78) stated that "the construction of the measuring devices is perhaps the most important segment of any study". Therefore, the PHMQ, in particular the frequency of maths activities scale, was evaluated across five psychometric properties (i.e. construct validity, factor structure, scale score reliability, content validity and criterion validity) and therefore satisfies APA standards for psychometric adequacy (APA, 1995; Hinkin, 1998). As with all questionnaire methods the PHMQ, is a self-report measure of the HNE and could be subject to social desirability bias. However, the PHMQ has been rigorously developed to allow researchers to obtain data efficiently to further understand how parents contribute to their preschool child's learning. Therefore, the PHMQ is a good measure to use with parents who have children between the ages of 3 and 5 as it is both developmentally appropriate and rigorously developed.

At this stage of the PHMQ development and validation, only one form of criterion validity has been included and no assessment of *predictive* validity has been reported. Due to the mixed findings in this area of research (Thompson et al, 2017) it is difficult to hypothesise what we would anticipate in terms of predictive validity. Thompson et al. (2017) examined studies relating HNE practices to mathematics performance and established that there are mixed findings in the literature. Some studies show positive directionality (i.e. Anders et al., 2012; Niklas, Cohrssen & Tayler, 2015), no significant relations (i.e. Belvins-Knabe et al., 2000; Missall et al., 2015) or indicate negative relations (i.e. Blevins-Knabe & Musun-Miller, 1996) between HNE practices and mathematics performance. In fact, both positive and null relationships (i.e. DeFlorio & Beliakoff, 2015; Zippert & Ramani, 2016) or both positive and negative relations (i.e. Skwarchuk, 2009) have been observed within the same study. Therefore, rather than focusing on the predictive nature of the PMHQ we aimed to generate a robustly developed measure with good construct validity, factor structure, scale score reliability, content validity and criterion validity. Thus, future research can utilse this measure to further assess if a relationship between the HME and mathematical development truly exists. Moreover, Daucourt's (2019) meta-analysis on the relationship between the HME and mathematics performance found, on average, a very small effect ($r = .14$). One of the major limitations of previous studies is that the measurement development process in these studies either 1) reference LeFevre et al. (2009) scale without further attention to age, cultural, or setting specific concerns or 2) present final items and only discuss internal consistency (e.g. Kleemans et al., 2012). In measurement development, reporting a clear and transparent outline of the process that was undertaken to generate the final measure is essential (Hinkin, 1998). One of the core contributions of the current study is that we focus on the measurement development process and provide a model that can be used in other contexts across the numerical cognition field.

One of the issues that may be driving the inconsistency of findings in this area, is the lack of agreement on how the HME should be defined. Our study has addressed this issue by defining the HME from the perspective of the parent through the first study of the four presented in this paper (also see the initial qualitative research to this project, Cahoon et al., 2017). Therefore, the main aim of this paper was to rigorously develop and validate a measure of the home environment that went right back to redefining the HME and subsequently demonstrating high levels of content and criterion validity.

Further, this study goes beyond only including frequency of maths activities question by including questions on children's *maths literacy and counting ability.* Additional dimensions/items were discovered and included such as *parent-child teaching methods* (e.g. what are the specific things parents say or do to encourage and support their child to learn maths?) and *target child-sibling interactions* (e.g. what numerical activities siblings are most likely to do together?). In addition to children interacting with their parents/caregivers at home, interactions with others, such as siblings, have been observed to play an important role in learning numerical concepts (Howe et al., 2015; Howe, Ross, & Recchia, 2011) however, these types of questions have rarely made it into HME questionnaires. These types of interaction questions could allow researchers to investigate if *parent-child teaching methods* and *target child-sibling interactions* help in the development of mathematical knowledge.

Limitations

Future research should attempt to align questionnaire measurement with other data collection techniques. This is particularly pertinent as the main focus of questionnaire based HNE measurement is the *frequency of activities.* Future studies should also attempt to measure *the quality of the content of these activities and interactions* which is a very difficult aspect to capture using questionnaires.

In both studies 2 and 4 there are more participants in the high SES category, with the middle SES category having the least participants. Hence, although considerable efforts were made to acquire an equal spread of participants across SES. There were less parents in the middle SES category and then the low SES category than the high SES category. However, this could be expected as research has shown that lower SES parents were less likely than others to engage in their child's schooling (e.g. Braun, Noden, Hind, McNally & West, 2005; Moon & Ivins, 2004; West, 2007).

Eight participant interviews were used in the criterion validity and although there were clear differences between the views and experiences of parents with low and high scores across all five PHMQ subscales the limited sample size used should be taken with caution. Further, it should be noted that the majority of items/questions used within the PHMQ are numeracy related which would be developmentally appropriate for the intended age group. However, the questionnaire involves home environment relevant dimensions beyond numeracy therefore it has been called the Pre-school Home *Mathematics* Questionnaire so as not to be misleading.

Future recommendations

Most HME questionnaires have been developed and used in home environments that reflect the developed world, for example Canada, America and the current PHMQ developed for a UK context. This is the first study within the UK that has created an HME questionnaire that is culturally specific, were items are not just deductive and drawn from other HME questionnaires such as Melhuish et al. (2008). Hence, this HME questionnaire alongside other available HME questionnaires may be context specific. There is a need for the development of an international measure that is developed and validated as rigorously as the current measure, but for the context of low-income country contexts. This study offered the theoretical and empirical framework of how an HME measure that reflects the home environment in low income countries could be created and validated to meet APA standards for psychometric adequacy (APA, 1995).

Conclusion

Some of the HME questionnaires have not provided adequate information about item generation and refinement, scale dimensionality, scale score reliability, or validity (e.g. Kleemans et al., 2012; LeFevre et al., 2009; Melhuish et al., 2008). In previous literature a major weakness to studying the HNE is the lack of information describing the psychometric integrity of scales used to measure the construct of the HNE. However, these studies have made a widespread impact on home learning environment research and the number of studies in this area have increased in recent years. The current study extends the rigour of HME questionnaire development and validation. This study provides details on psychometric integrity and appears to be psychometrically sound (Hinkin & Schriesheim, 1989; MacKenzie, Podsakoff & Fetter, 1991). The PHMQ covers a vast array of HNE areas thus, it is concluded that the PHMQ can be used to successfully describe the HNE that a parent creates for their child to learn numeracy. Every learning experience in the home are shared learning experiences for children, whether this is between parents or siblings. The PHMQ can allow researchers to quickly obtain data to understand how parents contribute to their child learning numeracy related concepts and skills.

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Supplementary information: Developing a Rigorous Measure of the Pre-school Home Mathematics Environment

Appendix 1. Pre-school Home Mathematics Questionnaire (PHMQ)

Instructions: Please complete the following questionnaire, answering all questions.
This questionnaire will take approximately 15 minutes to complete.
These questions are in relation to your child who is aged 3-4 years.
Please tick or circle the choice that best describes ***your family***.

ABOUT YOU

1. What age are you? ______________________________

2. What is your relationship to the participating child?

(a) Mother	
(b) Stepmother	
(c) Father	
(d) Stepfather	
(e) Grandparent	
(f) Foster parent	
(g) Adoptive parent	
(h) Other, please state:	

3. What is your current marital status?

(a) Single (never married)	
(b) Married	
(c) Cohabitating (not married)	
(d) Divorced	
(e) Separated	
(f) Widowed	

4. Are you the primary carer? (e.g. Spend most of the time with the child)

(a) Yes	
(b) No	

5. What is your ethnic origin?

(a) Asian	
(b) Black or African American	
(c) White, Caucasian	
(d) Chinese	
(e) Mixed	
(f) Other, please state:	

6. What is the first language you speak with your child?

(a) English	
(b) Irish	
(c) Spanish	
(d) French	
(e) Polish	
(f) Other, please state:	

7. What is your highest educational qualification?

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other, please state:	

8. What is your highest level of mathematical achievement? (Including any degree that involves statistical training)

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other, please state:	

9. Are you currently employed? **If <u>currently</u> employed proceed to question 12.**

(a) Yes full-time	
(b) Yes part-time	
(c) No	

10. If no, have you previously been employed? **If <u>previously</u> employed proceed to question 12.**

(a) Yes	
(b) No	

11. If no, do you provide full-time child-care? **If <u>full-time carer</u>, please proceed to question 20.**

(a) Yes	
(b) No	

<u>Details of current/previous employment</u>

12. What is/was your main job title?

____

13. What activities do/did you mainly do in your job?

____

14. What does/did the firm/organisation you worked for mainly make or do? (e.g. Provide leisure services, retail industry, education)

____

15. Are/were you working as an employee or are/were you self-employed?

(a) Employee		**If Employee – Go to question 16**
(b) Self-employed		**If Self-employed – Go to question 18**

Employee only

16. In your job, do/did you have any formal responsibility for supervising the work of other employees?

(a) Yes	
(b) No	

17. How many people work/worked for the employer at the place where you work/worked?

(a) 1 to 10		
(b) 11 to 24		
(c) 25 to 499		
(d) 500 or more employees		**Please continue to question 20**

Self-employed only

18. Are/were you working on your own or do/did you have employees?

(a) On own	
(b) With partner	
(c) No employees	
(d) Employees	
(e) Other:	

19. If you have/had employees, how many people do/did you employ at the place where you work/worked?

(a) 1 to 10	
(b) 11 to 24	
(c) 25 to 499	
(d) 500 or more employees	

OTHER ADULTS LIVING IN HOUSEHOLD

20. Are there other adults living in your household?

(a) Yes		**If Yes – please continue**
(b) No		**If No – Go to question 23**

21. Person’s relationship to child?

(a) Mother	
(b) Stepmother	

(c) Father	
(d) Stepfather	
(e) Grandparent	
(f) Foster parent	
(g) Adoptive parent	
(h) Other, please state:	

22. What is this adults highest educational qualification?

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other, please state:	

ABOUT YOUR PARTICIPATING CHILD

These questions are in relation to your child who is aged 3-4 years.

23. When was your child born? ____/____/______ (Day/Month/Year)

24. Including the child in question, how many children do you have in total?

Total number of children: ____________________

25. What is the birth order of your participating child aged 3 – 4?

(a) Only child	
(b) First born (oldest)	
(c) Second born	
(d) Third born	
(e) Fourth born	
(f) Fifth born	
(g) Other, please state:	

26. What is your participating child’s gender?

(a) Male	
(b) Female	

27. How many languages can your participating child speak?

(a) One	
(b) Two	
(c) Other, please state:	

28. What are these languages? ____

29. Are you interested in maths as a topic?

(a) No, I am completely disinterested in maths	
(b) No, I am disinterested in maths	
(c) Neither interested or disinterested in maths	
(d) Yes, I am interested in maths	
(e) Yes, I am very interested in maths	

30. Ideally, how much education would you want your participating child to complete?

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other, please state:	

31. Ideally, what would you want your participating child's highest mathematical achievement to be?

(a) GCSEs / O level / Irish Junior Certificate	
(b) A levels / BTEC / Irish Leaving Certificate	
(c) Degree	
(d) Masters	
(e) PhD	
(f) No qualifications	
(g) Other, please state:	

MATHS LITERACY

32. In the past month, how often did you and your child engage in reading? ***Please circle***

activity did not occur | few times a month | about once a week | few times a week | almost daily

33. Do any of the books you read to the participating child involve numbers?

(a) Yes	
(b) No	

If Yes – How many?
__________ (give as number)

34. Would you do maths activities more or less than reading?

(a) More	
(b) Less	
(c) Same	

NUMERACY

35. How high can your child currently count up to?

____

36. Did you ask your child to count to answer the above question?

(a) Yes	
(b) No	

37. How high do you think a child <u>at your child's age</u> should be able to count?

____

38. Who is more likely to bring up numeracy activities?

(a) You	
(b) Your child	
(c) Both	
(d) Other:	

39. Imagine you have asked your child a sum and they get the answer wrong, what are the specific things you say or do to encourage and support your child to learn maths?

Please order the following options in the order you would use each. ***1 - 'most likely' to 4 - 'least likely'*** ***Please do not leave any blank***	*Insert number below*	*Example:*
(a) Question and encourage your child without explanation (e.g. "No that's not the right answer, what number do you think it would be?")		1
(b) Prompt, explain and work through the problem together (e.g. Make sure he/she understand where they went wrong)		2
(c) Provide answer and move on		3
(d) Adjust your behaviour (e.g. demonstrate visually with		4

objects/fingers)		

FREQUENCY OF HOUSEHOLD ACTIVITIES

40. In the past month, how often did you and your child engage in the following? *Please circle*

1. Counting

activity did not occur | few times a month | about once a week | few times a week | almost daily

2. Write numbers

activity did not occur | few times a month | about once a week | few times a week | almost daily

3. Scenarios number games (e.g. "If I have two toy cars and I take one away, how many cars do I have?")

activity did not occur | few times a month | about once a week | few times a week | almost daily

4. Counting on fingers/hands

activity did not occur | few times a month | about once a week | few times a week | almost daily

5. Watching number related TV shows (e.g. Number Jacks or Numtums)

activity did not occur | few times a month | about once a week | few times a week | almost daily

6. Teaching about measurements (e.g. baking, height)

activity did not occur | few times a month | about once a week | few times a week | almost daily

7. Sticker books

activity did not occur | few times a month | about once a week | few times a week | almost daily

8. Sorting shapes

activity did not occur | few times a month | about once a week | few times a week | almost daily

9. Rhyming TV shows involving numbers (e.g. Number Jacks)

activity did not occur | few times a month | about once a week | few times a week | almost daily

10. Play with jigsaws

activity did not occur | few times a month | about once a week | few times a week | almost daily

11. Watch educational programs (e.g. Dora the Explorer)

activity did not occur | few times a month | about once a week | few times a week | almost daily

12. Sorting objects by size

activity did not occur | few times a month | about once a week | few times a week | almost daily

13. Comparing sets of objects (e.g. brother has more than mum)

activity did not occur | few times a month | about once a week | few times a week | almost daily

14. Pairing/matching games

activity did not occur | few times a month | about once a week | few times a week | almost daily

15. Playing with building blocks

activity did not occur | few times a month | about once a week | few times a week | almost daily

16. Identifying names of written numbers

activity did not occur | few times a month | about once a week | few times a week | almost daily

17. Counting out food, dinner plates, knifes and forks

activity did not occur | few times a month | about once a week | few times a week | almost daily

18. Creating patterns with objects (e.g. arranging blocks into shapes)

activity did not occur | few times a month | about once a week | few times a week | almost daily

19. Counting objects (e.g. ducks in bath, blocks, new toys, books)

activity did not occur | few times a month | about once a week | few times a week | almost daily

20. Teaching about money (e.g. informal – playing shop or formal – buying sweeties)

activity did not occur | few times a month | about once a week | few times a week | almost daily

21. Time terminology (e.g. big hand, little hand)

activity did not occur | few times a month | about once a week | few times a week | almost daily

22. Asking shape related questions (e.g. "how many sides does a circle have?")

activity did not occur | few times a month | about once a week | few times a week | almost daily

TECHNOLOGY

41. The following questions are all relating to technology usage (computers, tablets, smart phones). If your child **does not** use technology, please go to question 42.

In the past month, how often did your child engage in the following? ***Please circle***

1. Maths applications (e.g. Number Jacks)

activity did not occur | few times a month | about once a week | few times a week | almost daily

2. Maths related websites (e.g. coolmaths.com)

activity did not occur | few times a month | about once a week | few times a week | almost daily

3. Racing games (e.g. the faster they complete sums, the faster the boat moves)

activity did not occur | few times a month | about once a week | few times a week | almost daily

4. Size/matching apps (e.g. "put the big skirt on the small girl")

activity did not occur | few times a month | about once a week | few times a week | almost daily

5. Add and subtraction games

activity did not occur | few times a month | about once a week | few times a week | almost daily

6. Filling in the gap number games (e.g. what is next in the sequence?)

activity did not occur | few times a month | about once a week | few times a week | almost daily

7. Maths related YouTube videos (e.g. NumTums)

activity did not occur	few times a month	about once a week	few times a week	almost daily

BOARD GAMES

42. Below you will see a list of games for nursery children. Some of these are popular children's games, and some are made up.

<u>Please read the names and put a tick next to those games that you know to be real games. Do not guess, but only tick those you know.</u>

It is extremely important that you answer without stopping to verify any games.

(a) Battleships	
(b) Beach Shelter	
(c) Buckaroo	
(d) Build A Beetle	
(e) Chasin' Cheeky	
(f) Croc Doctor	
(g) Crocodile Dentist	
(h) Doctor Pop-up	
(i) Dog Tales	
(j) Doh Nutters Game	
(k) Dominoes	
(l) Elefun	
(m) Exasperation	
(n) Frustration	
(o) Guess who?	
(p) Head to toe	
(q) Hungry Hungry Hippo	
(r) Kerplunk	
(s) Ludo	
(t) Mailman	
(u) Mashup	
(v) Monopoly Junior	
(w) Operation	
(x) Pepper Pigs	

(y) Pie Face	
(z) Pop-up Pirate	
(aa) Shark Chase	
(bb) Snakes and Ladders	
(cc) Spider Web Master	
(dd) The Mashin Max Game	

SIBLINGS

43. Do you feel that your child has learnt number skills from their siblings?

(a) Yes	
(b) No	
(c) Does not apply	

44. When your children are doing activities together that involve maths, what types of activities are they most likely to do together? Keeping this in mind, in the past month, how often have you and your child engage in the following? ***Please circle***

1. Counting objects together

activity did not occur | few times a month | about once a week | few times a week | almost daily

2. Arranging objects by size, shape or colour

activity did not occur | few times a month | about once a week | few times a week | almost daily

3. Watching number related TV shows together (e.g. Number Jacks or Numtums)

activity did not occur | few times a month | about once a week | few times a week | almost daily

4. Sing rhyming songs together (e.g. “1, 2, 3, 4, 5 once I caught a fish alive”)

activity did not occur | few times a month | about once a week | few times a week | almost daily

5. Reading books together that involve numbers (e.g. Hungry Caterpillar)

activity did not occur | few times a month | about once a week | few times a week | almost daily

6. Timed games (e.g. hide and seek)

activity did not occur | few times a month | about once a week | few times a week | almost daily
|______________________|__________________|__________________|__________________|

7. Everyday activities that involve number (e.g. using money while shopping)

activity did not occur | few times a month | about once a week | few times a week | almost daily
|______________________|__________________|__________________|__________________|

Thank you for taking the time to fill in this questionnaire!

Appendix 2. Summary of Items from PHMQ

Table 1. Summary of Items, how they were Generated and Initial Item Reduction Criteria (before final PHMQ was created)

Qu No. from the original PHMQ	Items with home numeracy dimension category breakdown	Stage 1: Item Generation; Inductive or Deductive approach *	Stage 1: Deductive items overlap with literature	Stage 3: Initial Item Reduction; Kept or Removed **	Stage 3: Initial Item Reduction; Reason for removal
	Parent expectations – Benchmark questions				
30	Ideally, how much education would you want your participating child to complete?	Inductive		Kept	/
31	Ideally, what would you want your participating child's highest mathematical achievement to be?	Inductive		Kept	/
	Literacy – Benchmark questions				
32	In the past month, how often did you and your child engage in reading?	Deductive	LeFevre et al., 2009	Kept	/
33	Do any of the books you read to the participating child involve numbers?	Inductive		Kept	/
33a	If Yes – How many?	Inductive		Kept	/
34	Would you do maths activities more or less than reading?	Inductive		Kept	/
	Numeracy – Benchmark questions				
35	How high can your child currently count up to?	Deductive	LeFevre et al., 2009	Kept	/
36	Did you ask your child to count to answer the above question?	Deductive	LeFevre et al., 2009	Kept	/
37	How high do you think a child at your child's age should be able to count?	Inductive		Kept	/
	Parent-child interaction – Interaction questions				
38	Who is more likely to bring up numeracy activities?	Inductive		Kept	/
39	What are the specific things you say or do to encourage and support your child to learn maths?			Kept	/
39a	Question and encourage your child without explanation	Deductive	Vandermaas-Peeler et al., 2012	Kept	/

39b	Prompt, explain and work through the problem together	Deductive	Vandermaas-Peeler et al., 2012	Kept	/
39c	Provide answer and move on	Deductive	Vandermaas-Peeler et al., 2012	Kept	/
39d	Adjust your behaviour	Deductive	Vandermaas-Peeler et al., 2012	Kept	/
	Frequency of household activities				
40	In the past month, how often did you and your child engage in the following?				
1	Counting	Deductive	Melhuish et al., 2008	Kept	/
2	Feeding objects (e.g. posting letters)	Inductive		Removed	EFC
3	Hopscotch	Inductive		Removed	EFC
4	Write numbers	Deductive	LeFevre et al., 2009	Kept	/
5	Scenarios number games (e.g. "If I have two toy cars and I take one away, how many cars I have?")	Deductive	LeFevre et al., 2009; Lukie, Skwarchuk, LeFevre & Sowinski., 2014	Kept	/
6	Counting on fingers/hands	Inductive		Kept	/
7	Watching number related TV shows (e.g. Number Jacks or Numtums)	Inductive		Kept	/
8	Teaching about measurements (e.g. baking, height)	Deductive	LeFevre et al., 2009; Lukie, Skwarchuk, LeFevre & Sowinski., 2014	Kept	/
9	Sticker books	Inductive		Kept	/
10	Counting out turn taking (e.g. jumping to ten on trampoline)	Inductive		Removed	EFC
11	Sorting shapes	Deductive	LeFevre et al., 2009; Kleemans, Peeters, Segers & Verhoevena., 2012; Lukie,	Kept	/

			Skwarchuk, LeFevre & Sowinski., 2014		
12	Rhyming TV shows involving numbers (e.g. Number Jacks)	Inductive		Kept	/
13	Using number cards (e.g. order the cards by number)	Deductive	LeFevre et al., 2009; Lukie, Skwarchuk, LeFevre & Sowinski., 2014	Removed	EFC
14	Play with jigsaws	Inductive		Kept	/
15	Rhyming storybooks (e.g. Dr Seuss)	Inductive		Removed	EFC
16	Dot-to-dot number books	Deductive	LeFevre et al., 2009	Removed	EFC
17	Watch educational programs (e.g. Dora the Explorer)	Deductive	LeFevre et al., 2009	Kept	/
18	Sorting objects by size	Deductive	LeFevre et al., 2009	Kept	/
19	Counting up stairs	Inductive		Removed	EFC
20	Comparing sets of objects (e.g. brother has more than mum)	Inductive		Kept	/
21	Pairing/matching games	Inductive		Kept	/
22	Play card games (e.g. “jack change it”)	Deductive	LeFevre et al., 2009	Removed	EFC
23	Playing with building blocks	Deductive	LeFevre et al., 2009	Kept	/
24	Identifying names of written numbers	Deductive	LeFevre et al., 2009	Kept	/
25	Counting out food, dinner plates, knifes and forks	Inductive		Kept	/
26	Rhyming songs including counting (e.g. “1, 2, 3, 4, 5 once I caught a fish alive” or “ten green bottles”)	Deductive	Kleemans, Peeters, Segers & Verhoevena., 2012; Melhuish et al., 2008	Removed	EFC
27	Creating patterns with objects (e.g. arranging blocks into shapes)	Inductive		Kept	/

28	Being timed (e.g. hide and seek)	Deductive	LeFevre et al., 2009	Removed	EFC
29	Counting objects (e.g. ducks in bath, blocks, new toys, books)	Deductive	LeFevre et al., 2009	Kept	/
30	Teaching about money (e.g. informal – playing shop or formal – buying sweeties)	Deductive	LeFevre et al., 2009	Kept	/
31	Time terminology (e.g. big hand, little hand)	Deductive	Lukie, Skwarchuk, LeFevre & Sowinski., 2014	Kept	/
32	Asking shape related questions (e.g. “how many sides does a circle have?”)	Inductive		Kept	/
	Frequency of technology				
41	In the past month, how often did you and your child engage in the following?				
1	Maths applications (e.g. Number Jacks)	Inductive		Kept	/
2	Maths related websites (e.g. coolmaths.com)	Inductive		Kept	/
3	Racing games (e.g. faster they complete sums the faster the boat moves)	Inductive		Kept	/
4	Size/matching apps (e.g. “put the big skirt on the small girl”)	Inductive		Kept	/
5	Add and subtraction games	Inductive		Kept	/
6	Filling in the gap number games (e.g. what is next in the sequence?)	Inductive		Kept	/
	Siblings – Interaction questions				
42	Do you feel that your child has learnt number skills from their siblings?	Deductive	Benigno et al. (2004)	Kept	/
43	What would your participating child (aged 3 – 4) be more likely to do when engaged in a mathematical based activity with siblings?	Inductive		Removed	Lack of variation in responses
44	When your children are interacting mathematically, what types of activities are they most likely to do together?				
44a	Counting objects together	Inductive		Kept	/
44b	Arranging objects by size, shape or colour	Inductive		Kept	/
44c	Observing older siblings homework	Inductive		Removed	Lack of variance; Least likely to occur in the home

44d	Taking part in older siblings homework	Inductive	Removed	Lack of variance; Least likely to occur in the home
44e	Maths applications on technology device (e.g. Playing Number Jacks on iPhone)	Inductive	Removed	Lack of variance; Least likely to occur in the home
44f	Watching number related TV shows together (e.g. Number Jacks or Numtums)	Inductive	Kept	/
44g	Sing rhyming songs together (e.g. “1, 2, 3, 4, 5 once I caught a fish alive”)	Inductive	Kept	/
44h	Reading books together that involve numbers (e.g. Hungry Caterpillar)	Inductive	Kept	/
44i	Play board games or card games together (e.g. “jack change it”)	Inductive	Removed	Lack of variance; Least likely to occur in the home
44j	Timed games (e.g. hide and seek)	Inductive	Kept	/
44k	Everyday activities that involve number (e.g. using money while shopping)	Inductive	Kept	/
	Understanding			
45	Do you believe that your child understands the meaning of number words up to 5?	Inductive	Removed	Lack of variation in responses
46	Do you believe that your child understands the meaning of odd and even?	Inductive	Removed	Lack of variation in responses
47	Do you believe that your child understands the meaning of more and less?	Inductive	Removed	Lack of variation in responses
	Support			
48	Do you believe it is important for caregivers to support numeracy learning in the home?	Inductive	Removed	Lack of variation in responses

Note: * Inductive items = 44 items; Deductive items = 25; Total items = 69. ** Inductive items removed = 14; Deductive items removed = 5; Total items after removal = 50.

Appendix 3. Content Validity

Table 2. Content Validity: Dimensions (or Subscales) from PHMQ and Sample Commentary from Interviews

Dimension (or subscales) with definitions	Examples of items within the dimensions from PHMQ	Examples from interviews
Structure of the home numeracy environment		
Parent expectation of their children's academic success and child counting ability dimensions This theme is different in that it does not reflect one specific dimension of the PHMQ, moreover, it reflects two PHMQ dimensions and the balance between structured/formal and unstructured/informal numeracy environments. This was the same theme that was found in the original interviews (Cahoon et al., 2017)	How high can your child currently count up to? How high do you think a child at your child's age should be able to count? Who is more likely to bring up numeracy activities?	"He seems to be excelling at maths. He loves the counting and will do it himself now and he is only 3. I can hear him when he is on his own counting out figures, counting out Peppa Pig and separating things… even his Shreddies and Cheerio this morning for breakfast he counted those. So, we can nearly be counting all day without realising you're doing it, with nearly everything" – Participant 1 "When we are walking places it's easier to count things like how many red cars are there, so when you are out numeracy would be easier" – Participant 2 "I haven't gone out of my way to get numeracy games it's just everyday objects" – Participant 5
Child maths literacy		
Child maths literacy	In the past month, how often did you and your child engage in reading?	"If you're reading a book and if there is a picture, she'll say "Look mummy there's three dogs" or she'll count them "One, two, three" from the picture. There probably is more numeracy than literacy at the minute just because she is quite young" – Participant 2

The evidence suggests that children are accessing number learning through books.	Do any of the books you read to the participating child involve numbers? If so, how many? Would you do maths activities more or less than reading?	“Sometimes we would find things in books, sometimes I will say “find…” he is really into pirate so “find five swords in the picture”. In that instance I suppose there is that element of counting when he is searching for things, that’s quite frequent actually” – Participant 4 “She’ll count, she has a pirate book. When you get it right you push the button and it makes a little noise. The first page is ‘Count which arrow has four diamonds’ and there’s one arrow with 3 dots and an arrow with 4 diamonds. She counts the one with the dots 1, 2, 3 and then she will count the one with the 4 diamonds. They are very close together and she can still go 1, 2, 3, 4 and then she will press the button when she gets it right” – Participant 7
Parent-child teaching methods		
Parent-child teaching methods Each rank order option was mentioned in the current interviews.	Who is more likely to bring up numeracy activities? What are the specific interactions the parent/guardian would do to encourage and support the target child to learn numeracy? with 4 rank order options (e.g. question and encourage your child without explanation, prompt, explain and work through the problem together, provide answer and move on and adjust your behaviour).	“Lately it has been, “I’ve got 3 fingers on this hand and I’ve got 4 fingers on this hand, together that equals?”. Very showy stuff rather than in your head” – Participant 5 “Ella has these four dollies and I would say “Now Ella you have four dollies you could give two of those you Rob”. It’s working with items and visualising numbers, but practically as well” – Participant 6 “She loves counting, she’s really good at counting, she would count up to 20 and then I would try to do “One and add another one, what does that make?” (moved objects to demonstrate) but she’s not really getting it yet, she is too little” – Participant 7
	Target child-sibling interactions	
Target child-sibling interactions There were a wide variety of activities occurring between parent, target child, and older sibling/s, (i.e. triad interactions).	When your children are doing activities together that involve maths, what types of activities are they most likely to do together? Keeping this in mind, in the past month, how often have you and your child engage in the following?	“Her older brother is interested in maths. I would say she is maybe following his lead. She has an IKEA kitchen in the living room and I hear him counting sometimes. Then when he is at school I can hear her counting things just because that’s what he does” – Participant 2 “Most of the numeracy between the two of them would be about sharing. How much Rob has compared to how much she has and how to make it the same” – Participant 6

	Do you feel that your child has learnt number skills from their siblings?	"Amy is his older half-sister. They interact well considering the age gap. Amy would be very good, she would be a lot better than me, at going through things like colour. I would say she has taught Jake colours and she would go through the days of the week with him too" – Participant 8
Frequency of maths activities scale		
1. Parent –child interaction Any number-based interaction between the primary parent/guardian and their child in the home. Activities were a parent is necessary for the child to learn from the activity.	1. Write numbers 2. Scenarios number games (e.g. "If I have two toy cars and I take one away, how many cars I have?") 3. Teaching about measurements (e.g. baking, height) 4. Sticker books 5. Identifying names of written numbers 6. Teaching about money (e.g. informal – playing shop or formal – buying sweeties) 7. Time terminology (e.g. big hand, little hand) 8. Asking shape related questions (e.g. "how many sides does a circle have?")	"They'll (target child and older sibling) play together with Play Doh but there is usually a bit of a dispute if you leave them together alone. It's better if adults play with him than any of his peer group. He is still at the solidity play, well a bit of parallel play, but he's not moved onto co-operating" – Participant 1 "If we are baking I would try and get her to count the bun cases" – Participant 2 "She loves jigsaws. It's always supervised with mummy, and me going "You find another piece of Ariel's tail for me" but she loves it" – Participant 6
2. Computer maths games Any computer - based activities (such as, tablet or smartphone usage) that occur in the home, specifically games that involve number, shape or problem solving.	1. Maths applications (e.g. Number Jacks) 2. Maths related websites (e.g. coolmaths.com) 3. Racing games (e.g. faster they complete sums the faster the boat moves) 4. Size/matching apps (e.g. "put the big skirt on the small girl") 5. Add and subtraction games 6. Filling in the gap number games (e.g. what is next in the sequence?)	"On the iPad, he does the shadow into the shape, the racing games, and the one with the balloons on the number train" – Participant 1 "There's a Cbeebies app and the games on that are all educational" – Participant 2 "This EduKitchen app is good. So, there's a recycling bin and they pick up all the rubbish. They would have fruit and then wrappers to work out which ones go in the recycling bin so it is quite educational" – Participant 4
3. TV programmes	1. Watching number related TV shows (e.g. Number Jacks or Numtums) 2. Rhyming TV shows involving numbers (e.g. Number Jacks)	"TV can be a great motivator. You can say to them if we finish this then we'll put on Peppa Pig. It's great because they'll complete it before they go and watch TV" – Participant 1

Any educational TV programmes watched in the home involving rhymes and/or numbers.	3. Watch educational programmes (e.g. Dora the Explorer)	“He prefers cartoons but he does watch things like Mr Tumble and Gigglebiz. and there is Kerwhizz too. It’s a game show with aliens and ask number, shape or what’s missing questions” – Participant 4 “I’d rather them watch the Numtums (than non-educational TV), I think it’s quite good” – Participant 5
4. Shape Any shape, pattern or sorting based activity in the home.	1. Sorting shapes 2. Play with jigsaws 3. Sorting objects by size 4. Pairing/matching games 5. Playing with building blocks 6. Creating patterns with objects (e.g. arranging blocks into shapes)	He does the game with the wooden shapes, where you fit them into the holes and he loves matching cards like animal dominos where you match all the cows together” – Participant 4 “She’s good at jigsaws. She knows to do the straight edge, she’ll work from the corner. She has an 8 piece, 12 piece, 18 piece and a 24-piece jigsaw. She can do the 24 piece, she might need help. The smaller ones she can do on her own but the larger ones she’ll need a bit of help to get started” – Participant 7 “We would do puzzles together, jigsaws, and you can see his progression with more pieces now” - Participant 8
5. Counting Activities that involve the counting or comparing of objects in the home.	1. Counting 2. Counting on fingers/hands 3. Comparing sets of objects (e.g. brother has more than mum) 4. Counting out food, dinner plates, knives and forks 5. Counting objects (e.g. ducks in bath, blocks, new toys, books)	“She will count on her own without me prompting her. She’s very particular, almost an OCD level where everything has to be exact, she’s very exact when she comes to counting” – Participant 3 “He looks forward to his bedtime stories. In fairness, he gets to pick stories and now and again we’d say well you’ve been good so pick out 4 and he would go out and pick out 4 books. He picks out 2 books normally” – Participant 5 “She sits and count away to herself whenever she is playing, but she can only reliably count to 10 and then it becomes 33 and 54 and random numbers” – Participant 6

Appendix 4. Criterion Validity

Table 3. Criterion Validity: Subscale Dimensions and Sample Commentary from Interviews

Subscale dimension with definitions	Frequency items	Examples from interviews
1. Parent –child interaction	Low parent-child interaction	"He would help me bake now and again, not too often because of all the mess that comes with it but now and again he would help me cook and measure out ingredients" – Participant 5 (M = .63) "I suppose our main focus would be colours rather than numbers" – Participant 5 (M = .63) "It's not something that I have thought about (asking number-related questions while reading) but he got a homework book back, and there was a question in it about "what age do you think the girl is?" and he had to count the balloons. It wouldn't be something that I would have thought of" – Participant 5 (M = .63)
	High parent-child interaction	"We play with Play-Doh, rolling it up in balls, squashing it and counting it. This brings up counting and the shapes" – Participant 1 (M = 2.75) "We'll count the animals" (in a book they own at home) – Participant 1 (M = 2.75) "Obviously in the evening she's a bit tired and it's more fun rather than learning, and in the early afternoon when she's finished nursery we'll try and re-enforce what she has learnt that day whether it be the alphabet or numbers; any kind of homework" – Participant 3 (M = 2.38)
2. Computer maths games	Low computer maths games	"In this day and age there is more portable media and I worry how that would affect her learning. I think overuse of the game will affect her imagination, that creativity, that's why we limit it to maybe an hour a day at the very most" – Participant 3 (M = 1.00) "I prefer the games to be educational… The Edukitchen app is really good and the Cbeebies app is good too, because it makes him think. Furchester hotel as

		well. There is a problem he has to solve in each room and there are three ways he can solve the problem" – Participant 4 (M = .83)
	High computer maths games	"There is a Cbeebies app that I downloaded and it's for learning. He does colour in and counting activities on it" – Participant 8 (M = 2.00) "I would probably get half an hour's peace out of the Cbeebies app, whereas when he watches Batman on YouTube I would get an hour" – Participant 8 (M = 2.00)
3. TV programmes	Low TV programmes	Programmes with non – learning outcomes: "She likes a bit of My Little Pony but mostly Paw Patrol. Oh and Disney films, she loves Rapunzel and she loves Frozen" – Participant 6 (M = 1.00)
	High TV programmes	Programmes with learning outcomes: "She loves PJ masks, Peppa Pig, Lazy town, Numtums and Octonauts" – Participant 7 (M = 3.00) Programmes with learning outcomes: "The Cbeebies TV shows do have numbers because there's Numtums and Squiggle It too… I'd rather them watch Numtums and stuff like that. I think it's good for learning" – Participant 5 (M = 3.00)
4. Shape	Low shape	"I don't do that many structured activities. If they wanted to do painting or building blocks or do a jigsaw I would sit with them" – Participant 6 (M = 1.33) "He used to play jigsaws quite often before Rachel was born. He used to be very focused he would have sat and done a jigsaw and I actually thought he was quite smart at one point because he was doing the bigger jigsaws, bigger wooden ones that have 48 pieces" – Participant 5 (M = 1.67)
	High shape	"She loves Jenga. Jenga's her new favourite game. Sometimes we build houses with the Jenga block but she does quite like playing Jenga, pushing the blocks out. She's actually quite good at it" – Participant 2 (M = 3.00) "She loves puzzles. She loves jigsaws. She has lots of jigsaws" – Participant 2 (M = 3.00)

		"He's great at matching, we match beads and bags of pegs. Also in my sewing box we'll sort buttons into big, medium, small piles" – Participant 1 (M = 4.00) "To keep him engaged if you change the visual object, he thinks it's something new… he'll identify and sort out by colour and then he'll count" – Participant 1 (M = 4.00)
5. Counting	Low counting	"We would count the stairs and he would be counting along with me, but we've always been 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. It's just rote learning at the moment not sums" – Participant 5 (M = 1.60) "We count every day whether it be steps or how many things are in front of her because I do want her to start learning but it's probably a bit early to do like subtraction with her or anything like adding" – Participant 7 (M = 2.40)
	High counting	"She likes counting. If we are out somewhere, she will count flowers, or she will count dogs, like "They have two dogs."" – Participant 2 (M = 3.60) "She would count things spontaneously, be it if she's jumping up and down or the number of cows in a field, we live near a field, she'll count the cows' without me telling her too" – Participant 3 (M = 3.60) "She doesn't get as much time on her own as he did (older son) but she picks up a lot of things from him, so a lot of her spontaneous counting out objects is because he does it and she is copying him" – Participant 2 (M = 3.60)

Appendix 5. Confirmatory Factor Analysis for the Frequency of Maths Activities

Model one: One factor, total frequency of maths activities

Model one theorises a single factor construct consisting of all items measuring the frequency of maths activities. A one-factor model is also known as a g-factor or general factor model (Geiser, 2012). This model is based upon previous works that have measured the home numeracy environment through a unidimensional construct (Blevins-Knabe et al., 1996; Kleemans et al., 2012) where all activities occurring in the home environment related to maths development have been measured. This unidimensional approach provides a general overview of the influence of the home numeracy environment however, it does not give specific information or understanding of what types of activities and environments enhance early numeracy skills. The rationale for this model was to compare it against the five-factor model found through the exploratory factor analysis in the previous chapter to confirm which model was the superior fit.

Model three: Five-factor second order

Model three (Figure 1) is based on the concept that the five-factor model can be incorporated into two single factors; (1) *interaction with the parent*, constituted by the parent-child interaction, shape and counting subscales, and (2) *no interactions with the parent*, which includes computer maths games and TV programmes subscales. The three-factors, parent-child interaction, shape and counting were strongly correlated with each other (r = .77 to .88, Table 25, Figure 6). This indicates that there was little discriminant validity among the three-factors. Despite the strong correlations among the three-factors, the fit statistics indicated that more than one factor was needed to sufficiently account for the observed variances and covariance's of the five frequencies of numeracy-activities subscales and that a g-factor solution (Model one, Wang & Wang, 2012) had to be rejected for the current study. Even though the three-factors appear to share a substantial amount of common variance, there also is a nontrivial portion of systematic variance that needs to be taken into account and that explains why the correlations among the three-factors differ from 1.0 (Geiser, 2012).

Table 4. Pearson Zero-Order Correlations Between Subscales on the Frequency of Numeracy-Activities Scale (Model two: Five-Factor Model)

	1. Counting	2	3	4
2. Parent-child interaction	.881**			
3. TV programmes	.313**	.288*		
4. Shape	.785**	.773**	.381**	
5. Computer maths games	.283**	.481**	.437**	.411**

Note: * p < .05 ** p < .01 (two-tailed).

Thus, a five-factor second order model was theorised. This model hypotheses the use of a second order model, where the second order factors account for the variation among the first order factors (Wang & Wang, 2012). This model allows for the dimensionality of the frequency of numeracy-related activities to be explored further beyond the use of first order factors. In this model, three-factors in the five-factor model load onto a factor and the other two-factors in the five-factor model load onto another factor, so-called second order factors. One factor was given the name *interaction with the parent* as the activities in the parent-child interaction, shape and counting subscales could all involve interaction with the parent. The other two-factors, computer maths games and TV programmes, moderately correlated with each other (r = .44) and although this is not a strong enough correlation to assume that they are one second order factor it was deemed that these activities did not necessarily involve interactions with parents, as children are interacting with electronic devices. Thus, the two-factors, computer maths games and TV programmes, where named *no interaction with the parent* in the second order factor. Therefore, the five-factors are dichotomised into numeracy-related activities that could involve interactions with parents (parent-child interaction, shape and counting) and those activities that may not involve interactions with parents (computer maths games and TV programmes) in the second order model.

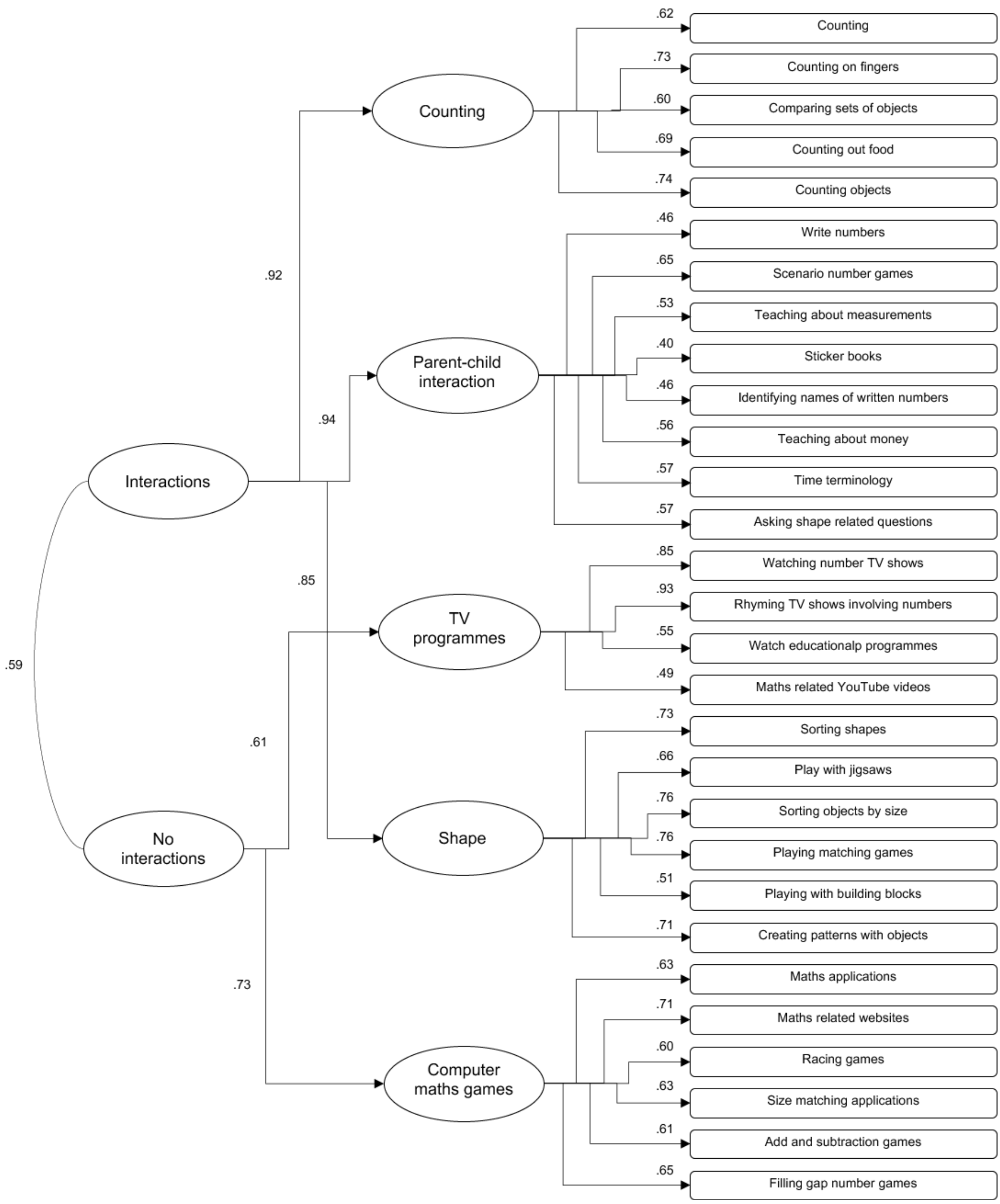

Figure 1. Five-Factor Second Order Model of the Frequency of Number Activities Scale in the PHMQ

Model four: Two-factor model based on the original definitions of direct and indirect numeracy activities by LeFevre et al. (2009)

Model four (Figure 2) is based on LeFevre et al. (2009) definitions of direct and indirect numeracy activities. The definitions set by LeFevre et al. (2009) suggested that *direct* activities, involving explicit and intentional teaching about numbers, quantity, or arithmetic to develop children's numeracy skills (e.g. counting objects) and *indirect* activities, involve numbers in real-world tasks (e.g. playing board games with dice) that include 'hidden' mathematical instructions that occur incidentally. LeFevre et al. (2009) found a four-factor structure, using a confirmatory factor analysis, which was labelled; (1) number skills (e.g. counting objects), (2) number books (e.g. "connect-the-dot" activities), (3) games (e.g. playing card games) and (4) applications (e.g. measuring ingredients when cooking). LeFevre et al. (2009) classified that those activities reported in the number skills and number books subscales reflected *direct* teaching activities and the games and application factors reflected *indirect* experiences. Thus, in the current study the researcher arbitrarily assigned the items into two subscales, direct and indirect numeracy activities to help with the selection of the best fitting model. The items assignment procedure for this model is included in Appendix 6.1 Items assignment procedure for Model 4, the items in each category can be seen in Figure 8. The rationale for this model was to compare it against the five-factor model hypothesised in the previous Chapter to help with the selection of the best fitting model.

Figure 2. Two-Factor Model Based on the Original Definitions by LeFevre et al. (2009)

Best fit model.

The selection of the most appropriate model was based upon goodness of fit statistics (see Table 3 in main manuscript). Models one and four failed to reach acceptable model fit with low CFI values .57 and TLI values .53. However, models two and three were the most acceptable model fit indices with each reporting a CFI of .81 and a TLI of .79. Good fitting models are indicated by a CFI of > .95 (better model: > .97) and the same cut-off value for TLI applies (Geiser, 2012). A CFI > .90 is often regarded as an indicator of an adequate model fit (Hair et al., 2010; Coroiu et al., 2018; Awang, 2012) the same cut-off value for TLI applies (Forza & Filippini, 1998; Coroiu et al., 2018; Awang, 2012).

The CFI and the TLI are incremental fit indices that compare the fit of the target model to the fit of a baseline model (Geiser, 2012). In Mplus the baseline model, also known as the null independence model, assumes that the population covariance matrix of the observed variables is a diagonal matrix, in other words it is assumed that there is no relationship between any of the variables (Geiser, 2012). As a consequence, it is possible that the null model is "too good", meaning that the average level of correlations in the current data is rather low. In this case, Kenny (2012) argued that CFI should not be computed if the RMSEA of the null model is less than 0.158 as the CFI obtained will be too small a value (Kenny & McCoach, 2003; Beldhuis, 2012). When investigating the RMSEA values both models two and three demonstrate acceptable RMSEA values (<0.08) (Awang, 2012), however model two has a lower RMSEA of .073 compared to model three that had a RMSEA of .074. Therefore, a slightly better RMSEA value was demonstrated for model two.

The SRMR coefficient is a standardised measure for the evaluation of the model residuals, however SRMR is somewhat biased by sample size. Marsh, Hau and Wen (2004) state that the SRMR values for solutions based on small sample sizes are unacceptable (greater than .08), whereas those based on large sample sizes are acceptable. A value < .08 is generally considered a good fit (Hu & Bentler, 1999). Therefore, given that model two presents the most acceptable fit indices (CFI = .81, TLI = .79, RMSEA = 0.073, SRMR = 0.078), it seems reasonable that a five-factor model, which includes parent-child interaction, computer maths games, TV programmes, shape and counting, to be deemed the most suitable measurement model. In order to obtain the best model, fit a closer examination of the factor loadings between models two and three was necessary. Slightly higher standardised factor loadings were demonstrated for model two than model three. With regards to the information criteria, BIC, AIC and adjusted BIC, model three has a smaller BIC (11753.563), but model two has a smaller AIC (11480.253) and ABIC (11455.925). Nevertheless, model two is the preferred model when taking into consideration all fit criteria for assessing goodness of fit.

Table 5. Model Fit Statistics for the Alternative Models of Frequency of Maths Activities (n = 136)

Model no.	Model explained	χ^2(p)	df	CFI	TLI	RMSEA (90% CI) p	SRMR	AIC	BIC	Sample-Size Adjusted BIC
1	One factor model	992.714 (0.00)	377	0.57	0.53	0.110 (0.101 – 0.118) 0.00	0.109	11841.537	12094.938	11819.717
2	Five-factor model	633.871 (0.00)	367	0.81	0.79	0.073 (0.063 – 0.083) 0.00	0.078	11480.253	11762.780	11455.925
3	Five-factor second order model	644.616 (0.00)	371	0.81	0.79	0.074 (0.064 – 0.083) 0.00	0.081	11482.686	11753.563	11459.361
4	Two-factor model (original direct and indirect activities)	992.703 (0.00)	376	0.57	0.53	0.110 (0.102 – 0.118) 0.00	0.109	11842.808	12099.121	11820.737

Note: Estimator = MLR; n = 136; *χ2* **=** Chi-square Goodness of Fit statistic; *df* = degrees of freedom; p = Statistical significance; CFI = Comparative Fit Index; TLI = Tucker Lewis Index; RMSEA (90% CI) = Root-Mean-Square Error of Approximation with 90% confidence intervals; BIC = Bayesian Information Criterion; AIC = Akaike information criterion