SM1

Derivations of Power and MDESD Formulas

Two-level CRTs with a Moderator at Level 2

We begin with a simple case that includes one treatment variable with equal allocation of clusters into the treatment and control groups and one level-2 moderator to illustrate basic concepts. As is typical in multilevel power analysis, we assume that the data are balanced such that each cluster has the same number of observations.

Equal Allocation, No Covariate Designs

The statistical power concerns the standard error of the moderator effect estimates. We start from reviewing the standard error estimate of a level-2 continuous predictor in a two-level hierarchical linear model (HLM) (Raudenbush and Bryk 2002).

The unconditional two-level HLM is:

Level 1:

$$Y_{ij} = \beta_{0j} + r_{ij},\ r_{ij} \sim N(0, \sigma^2) \tag{1}$$

Level 2:

$$\beta_{0j} = \gamma_{00} + u_{0j},\ u_{0j} \sim N(0, \tau^2) \tag{2}$$

The intra-class correlation (ICC) is defined as:

$$\rho = \tau^2 / (\tau^2 + \sigma^2)\,. \tag{3}$$

To estimate the moderator effect, we use a two-level hierarchical linear model. The level-1 model is the same as Expression (1). The level-2 model includes one treatment variable, T_j, coded as ±½, one level-2 continuous moderator, S_j, with grand mean centering, and the interaction term:

$$\beta_{0j} = \gamma_{00} + \gamma_{01}S_j + \gamma_{02}T_j + \gamma_{03}(S_j \times T_j) + u_{0j},\ u_{0j} \sim N(0, \tau^2_{|S,T}). \quad (4)$$

The parameter of interest for the moderator analysis is γ_{03}, which indicates the interaction/moderation effect.

We can represent the sample cluster means as:

$$\bar{Y}_{\cdot j} = \gamma_{00} + \gamma_{01}S_j + \gamma_{02}T_j + \gamma_{03}(S_j \times T_j) + (u_{0j} + \bar{r}_{\cdot j}),\ u_{0j} \sim N(0, \tau^2_{|S,T}),\ \bar{r}_{\cdot j} \sim N(0, \sigma^2 / n_j), \quad (5)$$

and the variance of $\bar{Y}_{\cdot j}$, given S_j, T_j, and $S_j \times T_j$ is

$$Var(\bar{Y}_{\cdot j}) = \tau^2_{|S,T} + \sigma^2 / n_j = \Delta^*_j, \quad (6)$$

where n_j is the sample size for level-2 cluster *j*.

When clusters have the same sample size (*n*), the Δ^*_j are identical in each group and the unique, minimum-variance, unbiased estimator of γ_{03} would be the ordinary least square (OLS) estimator (Raudenbush & Bryk, 2002), and the standard error of $\hat{\gamma}_{03}$ would be:

$$SE(\hat{\gamma}_{03}) = \sqrt{\frac{(1 - R^2_{\bar{Y},ST(S\times T)})\Delta_j}{(1 - R^2_{(S\times T),ST})(J-4)S^2_{S\times T}}}\ , \quad (7)$$

where Δ_j is the variance of $\bar{Y}_{\cdot j}$ in the unconditional model, $Var(\bar{Y}_{\cdot j}) = \tau^2 + \sigma^2 / n = \Delta_j$. $R^2_{\bar{Y},ST(S\times T)}$ is the proportion of variance in $\bar{Y}_{\cdot j}$ that is explained by S_j, T_j, and $S_j \times T_j$, i.e., $R^2_{\bar{Y},ST(S\times T)} = 1 - \frac{\Delta^*_j}{\Delta_j}$, $R^2_{(S\times T),ST}$ is the proportion of variance in $S_j \times T_j$ that is explained by S_j and T_j, and $S^2_{S\times T}$ is the variance of $S_j \times T_j$, *J* is the total number of clusters, and $(J-4)$ is the degree of freedom.

We can rewrite $R^2_{\bar{Y},ST(S\times T)}$ as a function of R^2_2, which is the proportion of variance at level 2 that is explained by the level-2 predictors (S_j, T_j, and $S_j \times T_j$), $R^2_2 = 1 - \frac{\tau^2_{|S,T}}{\tau^2}$. We will have:

$$R^2_{\bar{Y},ST(S\times T)} = 1 - \frac{\Delta_j^*}{\Delta_j} = 1 - \frac{\tau^2_{|S,T} + \sigma^2/n}{\tau^2 + \sigma^2/n} = 1 - \frac{(1-R_2^2)\tau^2 + \sigma^2/n}{\tau^2 + \sigma^2/n}. \quad (8)$$

In addition, when the treatment status is coded as ±½, and the level-2 continuous moderator, S_j, is the grand mean centered (i.e., $\bar{S}_j = 0$), three terms S_j, T_j, and $S_j \times T_j$ are independent. Hence, $R^2_{(S\times T),ST} = 0$, and $S^2_{S\times T} = S_S^2 \times S_T^2 + S_S^2 \times (\bar{T}_j)^2 + (\bar{S}_j)^2 \times S_T^2 = S_S^2 \times S_T^2$, where S_S^2 and S_T^2 are the variances of S_j and T_j.

The standard error of $\hat{\gamma}_{03}$ can be rewritten as:

$$SE(\hat{\gamma}_{03}) = \sqrt{\frac{\left(\frac{(1-R_2^2)\tau^2 + \sigma^2/n}{\tau^2 + \sigma^2/n}\right)(\tau^2 + \sigma^2/n)}{(J-4)S_S^2 S_T^2}} = \sqrt{\frac{(1-R_2^2)\tau^2 + \sigma^2/n}{(J-4)S_S^2 S_T^2}}, \quad (9)$$
$$= \sqrt{\frac{4[(1-R_2^2)\tau^2 + \sigma^2/n]}{(J-4)S_S^2}}$$

where S_T^2 = ¼ when it is a balanced design with equal allocation of clusters into the treatment and control groups.

We can test γ_{03} using a *t*-test. Assuming the alternative hypothesis is true, the test statistic follows a non-central *t*-distribution, *T'*. The noncentrality parameter (unstandardized) is a ratio of the moderator effect estimate to its standard error, as show in Equation A10:

$$\lambda_{|S} = \frac{\hat{\gamma}_{03}}{SE(\hat{\gamma}_{03})} = \frac{\hat{\gamma}_{03}}{\sqrt{\frac{4[(1-R_2^2)\tau^2 + \sigma^2/n]}{(J-4)S_S^2}}} = \sqrt{\frac{\hat{\gamma}_{03}^2 (J-4)S_S^2}{4[(1-R_2^2)\tau^2 + \sigma^2/n]}}. \quad (10)$$

We standardize the outcome (*Y*) and moderator (*S*), let $\tau^2 + \sigma^2 = 1$ and S_S^2 = 1, then $\hat{\gamma}_{03}$ is equal to the standardized coefficient δ_{2c}, or let $\delta_{2c} = \hat{\gamma}_{03}\sqrt{\frac{S_S^2}{\tau^2 + \sigma^2}}$, the noncentrality parameter (standardized) is:

$$\lambda_{|S} = \sqrt{\frac{\delta_{2c}^2 (J-4)}{4[(1-R_2^2)\rho + (1-\rho)/n)]}} \quad . \tag{11}$$

The statistical power for a two-sided test is (note $t_0 = t_{1-\alpha/2, J-4}$):

$1-\beta = 1 - P[T^{'}(J-4, \lambda_{|S}) < t_0] + P[T^{'}(J-4, \lambda_{|S}) \le -t_0]$, where the degrees of freedom is $v = J-4$, when the hierarchical linear model includes the treatment variable, moderator, and the interaction term for the treatment and moderator.

The minimum detectable effect size difference (MDESD) regarding the standardized coefficient is:

$$MDESD(|\delta_{2c}|) = M_v \frac{SE(\hat{\gamma}_{03})}{\tau^2 + \sigma^2} = M_v \sqrt{\frac{4[(1-R_2^2)\rho + (1-\rho)/n]}{J-4}} \, , \tag{12}$$

where, $M_v = t_\alpha + t_{1-\beta}$ for one-tailed tests with v degrees of freedom ($v = J-4$), and $M_v = t_{\alpha/2} + t_{1-\beta}$ for two-tailed tests.

The 100*(1-*α*)% confidence interval for $MDESD(|\delta_{2c}|)$ is given by:

$$(M_v \pm t_{\alpha/2}) \sqrt{\frac{4[(1-R_2^2)\rho + (1-\rho)/n]}{J-4}} \, . \tag{13}$$

Flexible Treatment Allocation, Covariate-controlled Designs

This is a more flexible design that allows unbalanced allocation of clusters to the treatment and control groups by defining *P* is the proportion of total clusters that are randomly assigned to the treatment group. In addition, we further expand the approach to allow a level-1 covariate, X_{ij} ($\bar{X}_{.j}$ is the group means), and a level-2 covariate, W_j. The two-level hierarchical linear model that generates data is in Expressions 1 and 2.

When we use the model (Expressions 1 and 2), we can represent the sample cluster means as:

$$\bar{Y}_{.j} = \gamma_{00} + \gamma_{01}S_j + \gamma_{02}T_j + \gamma_{03}(S_j \times T_j) + \gamma_{04}W_j + \gamma_{05}\bar{X}_{.j} + (u_{0j} + \bar{r}_{.j}),$$

$$u_{0j} \sim N(0, \tau^2_{|S,W,\bar{X},T}),\ \bar{r}_{.j} \sim N(0, \sigma^2_{|X}/n_j), \tag{14}$$

and the variance of $\bar{Y}_{.j}$, given S_j, T_j, $S_j \times T_j$, W_j, and $\bar{X}_{.j}$ is

$$Var(\bar{Y}_{.j}) = \tau^2_{|S,W,\bar{X},T} + \sigma^2_{|X}/n_j = \Delta_j^*, \tag{15}$$

where n_j is the sample size for level-2 cluster j.

Similar to the simple case, if every cluster had the same sample size (n), the standard error of the ordinary least square (OLS) estimator of $\hat{\gamma}_{03}$ (Raudenbush and Bryk 2002), is:

$$SE(\hat{\gamma}_{03}) = \sqrt{\frac{\left(1-R^2_{\bar{Y},ST(S\times T)WX}\right)\Delta_j}{\left(1-R^2_{(S\times T),STW\bar{X}}\right)(J-6)S^2_{S\times T}}}, \tag{16}$$

where $R^2_{\bar{Y},ST(S\times T)WX}$ is the proportion of variance in $\bar{Y}_{.j}$ that is explained by S_j, T_j, $S_j \times T_j$, W_j, and X_{ij}, i.e., $R^2_{\bar{Y},ST(S\times T)WX} = 1 - \frac{\Delta_j^*}{\Delta_j}$. $R^2_{(S\times T),STWX}$ is the proportion of variance in $S_j \times T_j$ that is explained by S_j, T_j, W_j, and $\bar{X}_{.j}$, and $S^2_{S\times T}$ is the variance of $S_j \times T_j$, J is the total number of clusters, and (J – 6) is the degree of freedom.

We can rewrite $R^2_{\bar{Y},ST(S\times T)WX}$ as a function of R_2^2, which is the proportion of variance at level 2 that is explained by the level-2 predictors (S_j, T_j, $S_j \times T_j$, $\bar{X}_{.j}$, and W_j), $R_2^2 = 1 - \frac{\tau^2_{|S,W,\bar{X},T}}{\tau^2}$, and R_1^2, which is the proportion of variance at level 1 that is explained by the level-1 predictor (X_{ij}), $R_1^2 = 1 - \frac{\sigma^2_{|X}}{\sigma^2}$. We will have:

$$R^2_{\bar{Y},ST(S\times T)WX} = 1 - \frac{\Delta_j^*}{\Delta_j} = 1 - \frac{\tau^2_{|S,W,\bar{X},T} + \sigma^2_{|X}/n}{\tau^2 + \frac{\sigma^2}{n}} = 1 - \frac{(1-R_2^2)\tau^2 + (1-R_1^2)\frac{\sigma^2}{n}}{\tau^2 + \frac{\sigma^2}{n}}. \tag{17}$$

In addition, when the treatment status is coded as ±½, and the level-2 continuous moderator, S_j, $\bar{X}_{.j}$, and the covariate, W_j, are the grand mean centered, $S_j \times T_j$ are independent of S_j, T_j, W_j, and $\bar{X}_{.j}$. Hence, $R^2_{(S\times T),STWX} = 0$, and $S^2_{S\times T} = S^2_S \times S^2_T + S^2_S \times (\bar{T}_j)^2 + (\bar{S}_j)^2 \times S^2_T = S^2_S \times S^2_T$.

The standard error of $\hat{\gamma}_{03}$ can be rewritten as:

$$SE(\hat{\gamma}_{03}) = \sqrt{\frac{\left(\frac{(1-R_2^2)\tau^2 + (1-R_1^2)\frac{\sigma^2}{n}}{\tau^2 + \frac{\sigma^2}{n}}\right)\left(\tau^2 + \frac{\sigma^2}{n}\right)}{(J-6)S_S^2 S_T^2}}$$

$$= \sqrt{\frac{(1-R_2^2)\tau^2+(1-R_1^2)\frac{\sigma^2}{n}}{(J-6)S_S^2 S_T^2}} = \sqrt{\frac{(1-R_2^2)\tau^2+(1-R_1^2)\frac{\sigma^2}{n}}{P(1-P)(J-6)S_S^2}}, \tag{18}$$

where S_S^2 is the variance of the moderator and S_T^2 is the variance of the treatment variable. S_T^2 $= P(1-P)$, where P is the proportion of clusters randomly assigned to the treatment group.

Similarly, we can test γ_{03} using a *t*-test (the degrees of freedom $v = J - 6$)[1], where the noncentrality parameter (unstandardized) is:

$$\lambda_{|S,W,X} = \frac{\hat{\gamma}_{03}}{SE(\hat{\gamma}_{03})} = \frac{\hat{\gamma}_{03}}{\sqrt{\frac{(1-R_2^2)\tau^2+(1-R_1^2)\frac{\sigma^2}{n}}{P(1-P)(J-6)S_S^2}}} = \sqrt{\frac{\hat{\gamma}_{03}^2 P(1-P)(J-6)S_S^2}{(1-R_2^2)\tau^2+(1-R_1^2)\sigma^2/n}}. \tag{19}$$

Following the same standardizing procedures as for the simple case in Expression A11, the noncentrality parameter (standardized) is:

$$\lambda_{|S,W,X} = \sqrt{\frac{\delta_{2c}^2 P(1-P)(J-6)}{(1-R_2^2)\rho+(1-R_1^2)(1-\rho)/n}}. \tag{20}$$

Extension to Binary Moderator

When S_j is a binary variable with a proportion of Q in one moderator subgroup and (1-Q)

[1] Generally, $v = J - g^* - 4$, where g^* is the number of Level 2 covariates (excluding the treatment variable, moderator, and moderator*treatment).

in another moderator subgroup, $S_j \sim Bernoulli(Q)$:

$$VAR(S_j) = S_S^2 = Q(1-Q)\,. \tag{21}$$

We insert Equation 21 into Equation 19, hence the standardized noncentrality parameters for the models with and without the group-mean centered level-1 covariate are:

$$\lambda_{|S,W,X} = \sqrt{\frac{\hat{\gamma}_{03}^2 P(1-P)(J-6)S_S^2}{(1-R_2^2)\tau^2+(1-R_1^2)\sigma^2/n}} = \sqrt{\frac{\delta_{2b}^2 P(1-P)Q(1-Q)(J-6)}{(1-R_2^2)\rho+(1-R_1^2)(1-\rho)/n}}, \tag{22}$$

where δ_{2b} is the effect size (standardized mean difference), $\delta_{2b} = \hat{\gamma}_{03}/\sqrt{\tau^2+\sigma^2}$.

Two-level CRTs with a Moderator at Level 1

For the randomly varying slope hierarchical linear model, including one treatment variable, T_j, and one level-1 moderator, S_{ij} ($S_{ij} \sim N(0, S_S^2)$), with random slope as in Expressions 7 and 8, the combined model is:

$$Y_{ij} = \gamma_{00} + \gamma_{10}S_{ij} + \gamma_{01}T_j + \gamma_{11}T_jS_{ij} + u_{1j}S_{ij} + u_{0j} + r_{ij}. \tag{23}$$

$$r_{ij} \sim N(0, \sigma_{|S}^2),\ \begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_{00|T}^2 & \tau_{01|T} \\ \tau_{10|T} & \tau_{11|T}^2 \end{pmatrix}\right).$$

For the control and treatment groups we have:

$$Y_{ij} = \gamma_{00} + \gamma_{10}S_{ij} + u_{1j}S_{ij} + u_{0j} + r_{ij} \tag{24}$$

and

$$Y_{ij} = \gamma_{00} + \gamma_{01} + (\gamma_{10} + \gamma_{11})S_{ij} + u_{1j}S_{ij} + u_{0j} + r_{ij}. \tag{25}$$

Based on the formula for the variance of the estimated regression coefficients of a level-1 variable with random slope (Snijders 2001, 2005), we have:

$$Var(\hat{\gamma}_{10}) = \frac{n\tau_{11|T}^2 S_S^2 + \sigma_{|S}^2}{JPnS_S^2} \tag{26}$$

and

$$Var(\hat{\gamma}_{10} + \hat{\gamma}_{11}) = \frac{n\tau_{11|T}^2 S_S^2 + \sigma_{|S}^2}{J(1-P)nS_S^2}, \tag{27}$$

where n is the number of level-1 units within each level-2 unit (cluster), J is the total number of clusters, and P is the proportion of clusters randomly assigned to the treatment group.

Because of random assignment, the control group and treatment group are independent, we have:

$$Var(\hat{\gamma}_{11}) = Var(\hat{\gamma}_{10} + \hat{\gamma}_{11} - \hat{\gamma}_{10}) = Var(\hat{\gamma}_{10} + \hat{\gamma}_{11}) + Var(\hat{\gamma}_{10})$$

$$= \frac{n\tau_{11|T}^2 S_S^2 + \sigma_{|S}^2}{J(1-P)nS_S^2} + \frac{n\tau_{11|T}^2 S_S^2 + \sigma_{|S}^2}{JPnS_S^2} = \frac{n\tau_{11|T}^2 S_S^2 + \sigma_{|S}^2}{JP(1-P)nS_S^2} = \frac{\tau_{11|T}^2 + \sigma_{|S}^2/(nS_S^2)}{JP(1-P)}. \quad (28)$$

Hence, the standard error of the parameter of interest (γ_{11}) is:

$$SE(\hat{\gamma}_{11}) = \sqrt{\frac{\tau_{11|T}^2 + \sigma_{|S}^2/(nS_S^2)}{P(1-P)J}} = \sqrt{\frac{(1-R_{2T}^2)\omega\tau_{00}^2 + (1-R_1^2)\sigma^2/(nS_S^2)}{P(1-P)J}}, \quad (29)$$

where σ^2 and τ_{00}^2 are the variances of residuals for level-1 and level-2 intercept in the unconditional model without any predictors. R_1^2 is the proportion of variance at level 1 that is explained by the level-1 moderator (X_{ij}): $R_1^2 = 1 - \frac{\sigma_{|S}^2}{\sigma^2}$. R_{2T}^2 is the proportion of the random slope (for S) variance explained by the treatment indicator (T_j): $R_{2T}^2 = 1 - \frac{\tau_{11|T}^2}{\tau_{11}^2}$. ω is the proportion of the variance (τ_{11}^2) between clusters on the effect of S_{ij} to the between-cluster residual variance (τ_{00}^2) when $\tau_{00}^2 > 0$ under the multilevel modeling framework, $\omega = \frac{\tau_{11}^2}{\tau_{00}^2}$. ω indicates the effect heterogeneity for the level-1 moderator (S_{ij}) across level-2 units (clusters) in the model that is not conditional on treatment variable, T_j.

We can test γ_{11} using a t-test. The noncentrality parameter (unstandardized) is:

$$\lambda_{|S} = \frac{\hat{\gamma}_{11}}{SE(\hat{\gamma}_{11})} = \frac{\hat{\gamma}_{11}}{\sqrt{\frac{(1-R_{2T}^2)\omega\tau_{00}^2 + (1-R_1^2)\sigma^2/(nS_S^2)}{P(1-P)J}}} \sqrt{\frac{\hat{\gamma}_{11}^2 P(1-P)J}{(1-R_{2T}^2)\omega\tau_{00}^2 + (1-R_1^2)\sigma^2/(nS_S^2)}} . \quad (30)$$

By standardization, let $\tau_{00}^2 + \sigma^2 = 1$ and $S_S^2 = 1$, the standardized coefficient $\delta_{1c} = \hat{\gamma}_{11}$, or let $\delta_{1c} = \hat{\gamma}_{11}\sqrt{\frac{S_S^2}{\tau_{00}^2 + \sigma^2}}$, the noncentrality parameter (standardized) is:

$$\lambda_{|S} = \sqrt{\frac{\delta_{1c}^2 P(1-P)J}{(1-R_{2T}^2)\rho\omega + (1-R_1^2)(1-\rho)/n}} . \quad (31)$$

The degrees of freedom is $\nu = J-2$. ρ is the unconditional intraclass correlation, $\rho = \frac{\tau_{00}^2}{\tau_{00}^2 + \sigma^2}$.

The statistical power for a two-sided test is (note $t_0 = t_{1-\alpha/2, J-2}$):
$1-\beta = 1 - P[T'(J-2, \lambda_{|S}) < t_0] + P[T'(J-2, \lambda_{|S}) \le -t_0]$.

The minimum detectable effect size difference (MDESD) regarding the standardized coefficient is:

$$MDESD(|\delta_{1c}|) = M_\nu \frac{SE(\hat{\gamma}_{11})}{\tau^2 + \sigma^2} = M_\nu \sqrt{\frac{(1-R_{2T}^2)\rho\omega + (1-R_1^2)(1-\rho)/n}{P(1-P)J}} , \quad (32)$$

where, $M_\nu = t_\alpha + t_{1-\beta}$ for one-tailed tests with ν degrees of freedom ($\nu = J-2$), and $M_\nu = t_{\alpha/2} + t_{1-\beta}$ for two-tailed tests.

The 100*(1-*α*)% confidence interval for $MDESD(|\delta_{1c}|)$ is given by:

$$(M_\nu \pm t_{\alpha/2})\sqrt{\frac{(1-R_{2T}^2)\rho\omega + (1-R_1^2)(1-\rho)/n}{P(1-P)J}} . \quad (33)$$

The hierarchical linear model with a nonrandomly varying slope assumes that the effect of S_{ij} varies by the treatment status (T_j), but does not vary across level-2 units (Expressions 7 & 12). Let ω = 0 in Expression A30, the unstandardized noncentrality parameter is:

$$\lambda_{|S} = \sqrt{\frac{\hat{\gamma}_{11}^2 P(1-P)JnS_S^2}{(1-R_1^2)\sigma^2}} \,. \tag{34}$$

The degrees of freedom[2] is $v = J(n-1) - 2$.

Using the same standardization procedure, we can get the standardized noncentrality parameter in Expression 13.

When the level-1 moderator, S_{ij}, is a binary variable with a proportion of Q in one moderator subgroup and (1 - Q) in another moderator subgroup, $S_{ij} \sim Bernoulli(Q)$:

$$VAR(S_{ij}) = \sigma_S^2 = Q(1-Q) \,. \tag{35}$$

We insert Expression A35 into Expressions A30 & A34, the noncentrality parameters (unstandardized) for the randomly varying slope model and the nonrandomly varying slope model are:

$$\lambda_{|S} = \sqrt{\frac{\hat{\gamma}_{11}^2 P(1-P)J}{(1-R_{2T}^2)\omega\tau_{00}^2 + (1-R_1^2)\sigma^2/(nQ(1-Q))}} \,, \tag{36}$$

and

$$\lambda_{|S} = \sqrt{\frac{\hat{\gamma}_{11}^2 P(1-P)Q(1-Q)Jn}{(1-R_1^2)\sigma^2}} \,. \tag{37}$$

By standardization, let $\delta_{1b} = \hat{\gamma}_{11} / \sqrt{\tau_{00}^2 + \sigma^2}$, the standardized noncentrality parameters for the randomly varying slope model and the nonrandomly varying slope model are:

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[2] Generally, $v = J(n-1) - 2 - g^*$, where g^* is the number of Level 1 covariates (excluding the moderator).

$$\lambda_{|S} = \sqrt{\frac{\delta_{1b}^2 P(1-P)J}{(1-R_{2T}^2)\rho\omega + (1-R_1^2)(1-\rho)/(nQ(1-Q))}}\,, \tag{38}$$

and

$$\lambda_{|S} = \sqrt{\frac{\delta_{1b}^2 P(1-P)Q(1-Q)Jn}{(1-R_1^2)(1-\rho)}}\,, \tag{39}$$

where ρ is the unconditional intraclass correlation, $\rho = \dfrac{\tau_{00}^2}{\tau_{00}^2 + \sigma^2}$.

SM2

Monte Carlo Simulation Procedures and Results

The procedures for the Monte Carlo simulation for a level-2 moderator are below:

(1) We generated data using the Hierarchical Linear Models in Expressions 1 and 2;

(2) We estimated the moderator effect, unconditional ICC, and proportions of variance (R^2) explained by level-1 and level-2 covariates using the estimation models in Expressions 1 and 2, and the unconditional Hierarchical Linear Models;

(3) The moderator effect was standardized to the effect sizes as standardized mean difference for the binary moderators or the standardized coefficient for the continuous moderators; a *p*-value of the moderator effect that is less than 0.05 was coded a rejection of the null hypothesis of no moderation;

(4) We replicated Steps (1) to (3) 10,000 times and calculated the means of the moderator effect size, unconditional ICC, and R^2; The proportion of times the null was rejected across the 10,000 replications estimated the Type I error rate when the moderation effect was 0 and the empirical power when the moderation effect was not 0; the standard deviation of 10,000 moderator effect sizes served as the standard error estimate based on the empirical distribution of the moderator effect; we also calculated the standard error based on our formulas, and constructed the 95% confidence interval (CI) for each point estimate; we calculated the absolute difference and relative difference between the standard errors based on our formulas and that from the empirical distribution; we calculate the coverage rate of the 95% CI as the percentage of the 95% CI based on our formulas covering the true moderator effect.

Our Monte Carlo simulation considered several scenarios by changing the sample size (J = 40 and n = 100; J = 120 and n = 20), unconditional ICC (ρ = 0.1 and 0.2), proportion of variance explained by the covariate at level 2 ($R_1^2 = R_2^2$ =0.4 and 0.7), the distribution of the moderator (binary and continuous), and the moderator effect size (0; 0.20) of the data generating models. There are 32 combinations of factor levels in total (8 for Type I error rate and 8 for statistical power for binary and continuous moderators, respectively).

We used SAS PROC MIXED to analyze the datasets. The simulation results provided evidence of the close correspondence on the standard error and power (or Type I error) between our formulas and the empirical distribution from the simulation (See Tables 1-4 in Electronic Supplementary Material 2). For example, in eight scenarios for a binary level-2 moderator the absolute difference and relative difference between the SE based on the empirical distribution of the moderator effect estimates and SE calculated from our formulas range from 0.001 to 0.021 and from 0.11% to 2.51%, respectively. The coverage rate of the 95% confidence interval (CI) range from 0.951 to 0.957. The Type I error rate estimated from simulation ranges from 0.047 to 0.054 while it is 0.05 based on our formulas. For other designs, the formulas also provided SE and power (or Type I error) estimates very close to those of the Monte Carlo simulation.

For a level-1 moderator, we conducted the Monte Carlo simulation using the similar procedures as CRT2-2. The effect heterogeneity (ω) for the level-1 moderator across level-2 units varied from 0 to 0.8. For each dataset, we used both the randomly varying slope model and the nonrandomly varying slope model to estimate the moderator effects. The simulation results provided evidence of the close correspondence on the standard error and power (or Type I error) between our formulas and the empirical distribution from the simulation when the analytic model was correctly specified (See Tables 5-8 in Electronic Supplementary Material 2).

Tables of Monte Carlo Simulation

Table S1

Coverage of 95% Confidence Interval and Type I Error Rate from Monte Carlo Simulation and the Formulas for a Binary Moderator in CRT2-2 Model

Scenario	1	2	3	4	5	6	7	8
ρ	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1
R_1^2	0.7	0.4	0.7	0.4	0.7	0.4	0.7	0.4
R_2^2	0.7	0.4	0.7	0.4	0.7	0.4	0.7	0.4
n (Average Cluster Size)	20	20	100	100	20	20	100	100
J (# of Clusters)	100	100	40	40	100	100	40	40
SE based on empirical distribution	0.447	0.825	0.689	1.276	0.480	0.902	0.698	1.318
SE calculated from formula	0.447	0.833	0.697	1.297	0.492	0.918	0.714	1.330
Absolute difference in SE	0.000	0.008	0.007	0.021	0.012	0.015	0.016	0.012
Relative difference in SE (%)	0.11	1.00	1.07	1.66	2.51	1.71	2.33	0.88
Coverage rate of 95% CI	0.951	0.952	0.955	0.957	0.955	0.956	0.955	0.954
Type I error rate estimated from simulation	0.053	0.051	0.053	0.048	0.048	0.047	0.051	0.054
Type I error rate calculated from formulas	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
Absolute difference in Type I error rate	-0.003	-0.001	-0.003	0.002	0.003	0.003	-0.001	-0.003
Relative difference in Type I error rate (%)	-5.83	-1.38	-6.14	3.52	5.27	7.31	-2.72	-6.51

Note. Results were based on 10,000 replications. ρ is the intraclass correlation. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_2^2 is the proportion of variance at level 2 explained by covariates. The effect size difference was set as 0. The proportion of clusters assigned to the treatment group, $P = 0.5$. The proportion of the clusters in one moderator subgroup, $Q = 0.5$. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at $\alpha = 0.05$. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S2

Coverage of 95% Confidence Interval and Power from Monte Carlo Simulation and the Formulas for a Binary Moderator in CRT2-2 Model

Scenario	1	2	3	4	5	6	7	8
ρ	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1
R_1^2	0.7	0.4	0.7	0.4	0.7	0.4	0.7	0.4
R_2^2	0.7	0.4	0.7	0.4	0.7	0.4	0.7	0.4
n (Average Cluster Size)	20	20	100	100	20	20	100	100
J (# of Clusters)	100	100	40	40	100	100	40	40
SE based on empirical distribution	0.449	0.837	0.687	1.298	0.493	0.936	0.718	1.346
SE calculated from formula	0.452	0.847	0.703	1.320	0.503	0.946	0.730	1.370
Absolute difference in standard errors	0.003	0.010	0.016	0.022	0.010	0.010	0.012	0.024
Relative difference in standard errors (%)	0.63	1.22	2.28	1.70	2.02	1.02	1.65	1.79
Coverage rate of 95% CI	0.952	0.956	0.956	0.957	0.955	0.952	0.957	0.958
Power estimated from simulation	0.433	0.241	0.213	0.132	0.661	0.355	0.363	0.191
Power calculated from formulas	0.424	0.237	0.207	0.129	0.648	0.348	0.349	0.189
Absolute difference in power	-0.009	-0.004	-0.005	-0.003	-0.012	-0.007	-0.015	-0.002
Relative difference in power (%)	-2.17	-1.81	-2.51	-2.62	-1.89	-2.00	-4.04	-0.88

Note. Results were based on 10,000 replications. ρ is the intraclass correlation. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_2^2 is the proportion of variance at level 2 explained by covariates. The effect size difference was set as 0.2. The proportion of clusters assigned to the treatment group, $P = 0.5$. The proportion of the clusters in one moderator subgroup, $Q = 0.5$. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at $\alpha = 0.05$. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S3

Coverage of 95% Confidence Interval and Type I Error Rate from Monte Carlo Simulation and the Formulas for a Continuous Moderator in CRT2-2 Model

Scenario	1	2	3	4	5	6	7	8
ρ	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1
R_1^2	0.7	0.4	0.7	0.4	0.7	0.4	0.7	0.4
R_2^2	0.7	0.4	0.7	0.4	0.7	0.4	0.7	0.4
n (Average Cluster Size)	20	20	100	100	20	20	100	100
J (# of Clusters)	120	120	40	40	120	120	40	40
SE based on empirical distribution	0.344	0.646	0.540	1.024	0.376	0.699	0.554	1.047
SE calculated from formula	0.341	0.636	0.530	0.988	0.376	0.701	0.544	1.013
Absolute difference in standard errors	-0.003	-0.010	-0.010	-0.036	0.000	0.002	-0.010	-0.034
Relative difference in standard errors (%)	-0.77	-1.53	-1.88	-3.53	0.12	0.26	-1.74	-3.29
Coverage rate of 95% CI	0.951	0.948	0.949	0.945	0.947	0.949	0.946	0.945
Type I error rate estimated from simulation	0.049	0.051	0.049	0.053	0.053	0.051	0.050	0.053
Type I error rate calculated from formula	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
Absolute difference in Type I error rate	0.001	-0.001	0.001	-0.003	-0.003	-0.001	0.000	-0.003
Relative difference in Type I error rate (%)	1.22	-2.46	2.46	-5.83	-4.92	-1.95	-0.18	-6.13

Note. Results were based on 10,000 replications. ρ is the intraclass correlation. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_2^2 is the proportion of variance at level 2 explained by covariates. The effect size difference was set as 0. The proportion of clusters assigned to the treatment group, P = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S4

Coverage of 95% Confidence Interval and Power from Monte Carlo Simulation and the Formulas for a Continuous Moderator in CRT2-2 Model

Scenario	1	2	3	4	5	6	7	8
ρ	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1
R_1^2	0.7	0.4	0.7	0.4	0.7	0.4	0.7	0.4
R_2^2	0.7	0.4	0.7	0.4	0.7	0.4	0.7	0.4
n (Average Cluster Size)	20	20	100	100	20	20	100	100
J (# of Clusters)	120	120	40	40	120	120	40	40
SE based on empirical distribution	0.356	0.683	0.558	1.070	0.403	0.806	0.593	1.186
SE calculated from formula	0.354	0.680	0.551	1.058	0.406	0.810	0.589	1.175
Absolute difference in standard errors	-0.003	-0.003	-0.007	-0.012	0.004	0.005	-0.004	-0.011
Relative difference in standard errors (%)	-0.73	-0.37	-1.17	-1.10	0.87	0.56	-0.66	-0.94
Coverage rate of 95% CI	0.949	0.952	0.950	0.948	0.954	0.953	0.948	0.949
Power estimated from simulation	0.933	0.696	0.599	0.339	0.995	0.897	0.858	0.589
Power calculated from formula	0.938	0.697	0.599	0.346	0.994	0.899	0.861	0.593
Absolute difference in power	0.005	0.002	0.000	0.007	0.000	0.002	0.003	0.004
Relative difference in power (%)	0.57	0.24	-0.08	1.92	-0.02	0.20	0.35	0.71

Note. Results were based on 10,000 replications. ρ is the intraclass correlation. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_2^2 is the proportion of variance at level 2 explained by covariates. The effect size difference was set as 0.2. The proportion of clusters assigned to the treatment group, $P = 0.5$. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at $\alpha = 0.05$. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S5

Coverage of 95% Confidence Interval and Type I Error Rate from Monte Carlo Simulation and the Formulas for a Binary Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0 in the Data Generation Model

Scenario	1		2		3		4	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes
ω	.	0.05	.	0.08	.	0.04	.	0.06
R_1^2	0.70	0.70	0.40	0.40	0.70	0.70	0.40	0.40
R_{2T}^2	.	0.31	.	0.25	.	0.30	.	0.26
n (Average Cluster Size)	20	20	20	20	40	40	40	40
J (# of Clusters)	40	40	40	40	20	20	20	20
SE based on empirical distribution	0.072	0.072	0.103	0.103	0.073	0.073	0.102	0.102
SE calculated from formula	0.069	0.074	0.098	0.103	0.069	0.076	0.098	0.106
Absolute difference in standard errors	-0.002	0.002	-0.005	0.001	-0.004	0.003	-0.004	0.004
Relative difference in standard errors (%)	-0.03	0.03	-0.04	0.01	-0.05	0.04	-0.04	0.04
Coverage rate of 95% CI	0.944	0.949	0.944	0.950	0.943	0.950	0.949	0.957
Type I error rate estimated from simulation	0.051	0.043	0.051	0.045	0.054	0.045	0.049	0.041
Type I error rate calculated from formula	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
Absolute difference in Type I error rate	-0.001	0.007	-0.001	0.006	-0.004	0.005	0.001	0.009
Relative difference in Type I error rate (%)	-1.96	16.01	-2.11	12.40	-7.40	10.87	1.76	23.28

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The effect size difference = 0. The intraclass correlation, ρ = 0.2. The proportion of clusters assigned to the treatment group, P = 0.5. The proportion of the individuals in one moderator subgroup, Q = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S6

Coverage of 95% Confidence Interval and Power from Monte Carlo Simulation and the Formulas for a Binary Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0 in the Data Generation Model

Scenario	1		2		3		4		5		6		7		8	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Effect Size Difference	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10
ω	.	0.09	.	0.11	.	0.06	.	0.08	.	0.08	.	0.09	.	0.05	.	0.07
R_1^2	0.70	0.70	0.39	0.40	0.70	0.70	0.40	0.40	0.70	0.70	0.39	0.40	0.70	0.70	0.40	0.40
R_{2T}^2	.	0.46	.	0.36	.	0.35	.	0.28	.	0.52	.	0.41	.	0.36	.	0.30
n (Average Cluster Size)	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40
J (# of Clusters)	40	40	40	40	40	40	40	40	20	20	20	20	20	20	20	20
SE based on empirical distribution	0.072	0.072	0.102	0.102	0.072	0.072	0.102	0.102	0.072	0.072	0.103	0.103	0.072	0.072	0.101	0.101
SE calculated from formula	0.069	0.076	0.098	0.105	0.069	0.075	0.098	0.104	0.069	0.079	0.098	0.109	0.069	0.077	0.098	0.107
Absolute difference in standard errors	-0.002	0.004	-0.004	0.003	-0.002	0.003	-0.004	0.002	-0.003	0.007	-0.005	0.006	-0.003	0.005	-0.003	0.006
Relative difference in standard errors (%)	-0.03	0.05	-0.04	0.03	-0.03	0.04	-0.04	0.02	-0.04	0.10	-0.05	0.06	-0.04	0.07	-0.03	0.05
Coverage rate of 95% CI	0.946	0.951	0.947	0.952	0.946	0.952	0.945	0.952	0.948	0.957	0.944	0.953	0.947	0.955	0.949	0.957
Power estimated from simulation	0.795	0.769	0.508	0.479	0.282	0.260	0.167	0.149	0.787	0.750	0.504	0.460	0.283	0.249	0.164	0.139
Power calculated from formula	0.820	0.753	0.535	0.483	0.299	0.265	0.175	0.161	0.817	0.711	0.534	0.456	0.306	0.256	0.176	0.156
Absolute difference in power	0.025	-0.016	0.027	0.004	0.017	0.006	0.008	0.013	0.030	-0.039	0.030	-0.004	0.023	0.008	0.012	0.017
Relative difference in power (%)	3.10	-2.13	5.36	0.79	6.14	2.27	4.97	8.51	3.83	-5.23	5.99	-0.90	8.14	3.16	7.35	12.45

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The intraclass correlation, $\rho = 0.2$. The proportion of clusters assigned to the treatment group, $P = 0.5$. The proportion of the individuals in one moderator subgroup, $Q = 0.5$. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at $\alpha = 0.05$. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S7

Coverage of 95% Confidence Interval and Type I Error Rate from Monte Carlo Simulation and the Formulas for a Continuous Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0 in the Data Generation Model

Scenario	1		2		3		4	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes
ω	.	0.01	.	0.02	.	0.01	.	0.01
R_1^2	0.70	0.70	0.40	0.40	0.70	0.70	0.40	0.40
R_{2T}^2	.	0.30	.	0.25	.	0.30	.	0.25
n (Average Cluster Size)	20	20	20	20	40	40	40	40
J (# of Clusters)	40	40	40	40	20	20	20	20
SE based on empirical distribution	0.037	0.037	0.051	0.051	0.036	0.036	0.052	0.052
SE calculated from formula	0.035	0.037	0.049	0.052	0.035	0.038	0.049	0.053
Absolute difference in standard errors	-0.002	0.000	-0.002	0.001	-0.001	0.002	-0.003	0.001
Relative difference in standard errors (%)	-0.06	0.01	-0.03	0.01	-0.04	0.05	-0.05	0.03
Coverage rate of 95% CI	0.936	0.941	0.944	0.949	0.950	0.955	0.948	0.955
Type I error rate estimated from simulation	0.057	0.050	0.052	0.044	0.046	0.039	0.049	0.041
Type I error rate calculated from formula	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
Absolute difference in Type I error rate	-0.007	0.000	-0.002	0.006	0.004	0.011	0.001	0.009
Relative difference in Type I error rate (%)	-11.95	-0.37	-3.47	13.90	7.77	27.57	2.69	22.88

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The effect size difference = 0. The intraclass correlation, ρ = 0.2. The proportion of clusters assigned to the treatment group, P = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S8

Coverage of 95% Confidence Interval and Power from Monte Carlo Simulation and the Formulas for a Continuous Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0 in the Data Generation Model

Scenario	1		2		3		4		5		6		7		8	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Effect Size Difference	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10
ω	.	0.05	.	0.06	.	0.02	.	0.03	.	0.06	.	0.06	.	0.02	.	0.02
R_1^2	0.70	0.70	0.40	0.40	0.70	0.70	0.40	0.40	0.70	0.70	0.40	0.40	0.70	0.70	0.40	0.40
R_{2T}^2	.	0.80	.	0.66	.	0.46	.	0.37	.	0.86	.	0.74	.	0.52	.	0.41
n (Average Cluster Size)	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40
J (# of Clusters)	40	40	40	40	40	40	40	40	20	20	20	20	20	20	20	20
SE based on empirical distribution	0.036	0.036	0.051	0.052	0.037	0.037	0.051	0.052	0.037	0.037	0.051	0.051	0.036	0.037	0.051	0.051
SE calculated from formula	0.035	0.038	0.049	0.053	0.035	0.038	0.049	0.052	0.035	0.039	0.049	0.055	0.035	0.040	0.049	0.054
Absolute difference in standard errors	-0.001	0.002	-0.002	0.001	-0.002	0.001	-0.002	0.001	-0.002	0.002	-0.002	0.003	-0.002	0.003	-0.002	0.003
Relative difference in standard errors (%)	-0.03	0.05	-0.05	0.02	-0.05	0.03	-0.05	0.01	-0.05	0.07	-0.04	0.07	-0.05	0.08	-0.04	0.06
Coverage rate of 95% CI	0.946	0.953	0.940	0.947	0.941	0.946	0.942	0.949	0.946	0.953	0.948	0.954	0.941	0.949	0.947	0.954
Power estimated from simulation	1.000	1.000	0.973	0.966	0.784	0.758	0.503	0.473	0.999	0.999	0.975	0.962	0.780	0.744	0.493	0.449
Power calculated from formula	1.000	1.000	0.983	0.966	0.818	0.750	0.536	0.485	1.000	0.999	0.982	0.954	0.817	0.710	0.524	0.448
Absolute difference in power	0.000	0.000	0.010	0.000	0.034	-0.008	0.033	0.012	0.001	0.000	0.008	-0.008	0.037	-0.034	0.031	0.000
Relative difference in power (%)	0.00	-0.01	1.08	0.02	4.35	-1.00	6.47	2.52	0.05	0.04	0.81	-0.80	4.78	-4.59	6.23	-0.11

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The intraclass correlation, ρ = 0.2. The proportion of clusters assigned to the treatment group, *P* = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S9

Coverage of 95% Confidence Interval and Type I Error Rate from Monte Carlo Simulation and the Formulas for a Binary Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.2 in the Data Generation Model

Scenario	1		2		3		4	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes
ω	.	0.22	.	0.22	.	0.23	.	0.23
R_1^2	0.69	0.70	0.39	0.40	0.69	0.70	0.39	0.40
R_{2T}^2	.	0.04	.	0.05	.	0.05	.	0.07
n (Average Cluster Size)	20	20	20	20	40	40	40	40
J (# of Clusters)	40	40	40	40	20	20	20	20
SE based on empirical distribution	0.096	0.096	0.119	0.120	0.116	0.116	0.138	0.138
SE calculated from formula	0.071	0.095	0.099	0.117	0.071	0.116	0.099	0.135
Absolute difference in standard errors	-0.026	-0.002	-0.021	-0.002	-0.046	0.000	-0.039	-0.003
Relative difference in standard errors (%)	-0.27	-0.02	-0.17	-0.02	-0.39	0.00	-0.28	-0.02
Coverage rate of 95% CI	0.856	0.946	0.902	0.949	0.772	0.952	0.851	0.947
Type I error rate estimated from simulation	0.133	0.058	0.090	0.054	0.222	0.062	0.145	0.067
Type I error rate calculated from formula	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
Absolute difference in Type I error rate	-0.083	-0.008	-0.040	-0.004	-0.172	-0.012	-0.095	-0.017
Relative difference in Type I error rate (%)	-62.29	-13.34	-44.42	-7.14	-77.50	-19.73	-65.52	-24.92

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The effect size difference = 0. The intraclass correlation, ρ = 0.2. The proportion of clusters assigned to the treatment group, P = 0.5. The proportion of the individuals in one moderator subgroup, Q = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S10

Coverage of 95% Confidence Interval and Power from Monte Carlo Simulation and the Formulas for a Binary Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.2 in the Data Generation Model

Scenario	1		2		3		4		5		6		7		8	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Effect Size Difference	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10
ω	.	0.22	.	0.22	.	0.22	.	0.22	.	0.23	.	0.23	.	0.23	.	0.23
R_1^2	0.69	0.70	0.39	0.40	0.69	0.70	0.39	0.40	0.69	0.70	0.39	0.40	0.69	0.70	0.39	0.40
R_{2T}^2	.	0.27	.	0.26	.	0.09	.	0.10	.	0.26	.	0.27	.	0.10	.	0.11
n (Average Cluster Size)	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40
J (# of Clusters)	40	40	40	40	40	40	40	40	20	20	20	20	20	20	20	20
SE based on empirical distribution	0.091	0.091	0.117	0.117	0.095	0.095	0.117	0.117	0.109	0.109	0.128	0.128	0.114	0.114	0.136	0.136
SE calculated from formula	0.070	0.089	0.099	0.113	0.071	0.093	0.099	0.117	0.070	0.108	0.099	0.128	0.071	0.114	0.099	0.133
Absolute difference in standard errors	-0.021	-0.002	-0.018	-0.003	-0.025	-0.002	-0.018	0.000	-0.039	-0.002	-0.029	0.000	-0.043	0.000	-0.037	-0.002
Relative difference in standard errors (%)	-0.23	-0.03	-0.15	-0.03	-0.26	-0.02	-0.15	0.00	-0.35	-0.01	-0.23	0.00	-0.38	0.00	-0.27	-0.02
Coverage rate of 95% CI	0.875	0.945	0.905	0.947	0.860	0.946	0.905	0.951	0.806	0.942	0.880	0.955	0.787	0.950	0.861	0.947
Power estimated from simulation	0.732	0.592	0.490	0.406	0.332	0.192	0.202	0.138	0.696	0.475	0.498	0.355	0.360	0.158	0.238	0.124
Power calculated from formula	0.809	0.590	0.518	0.401	0.294	0.182	0.174	0.134	0.809	0.424	0.531	0.323	0.288	0.131	0.174	0.111
Absolute difference in power	0.077	-0.002	0.028	-0.004	-0.039	-0.010	-0.028	-0.004	0.113	-0.051	0.033	-0.031	-0.072	-0.027	-0.064	-0.013
Relative difference in power (%)	10.52	-0.25	5.74	-1.10	-11.63	-5.04	-13.96	-2.93	16.18	-10.73	6.60	-8.83	-20.06	-17.06	-26.87	-10.81

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The intraclass correlation, $\rho = 0.2$. The proportion of clusters assigned to the treatment group, $P = 0.5$. The proportion of the individuals in one moderator subgroup, $Q = 0.5$. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at $\alpha = 0.05$. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S11

Coverage of 95% Confidence Interval and Type I Error Rate from Monte Carlo Simulation and the Formulas for a Continuous Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.2 in the Data Generation Model

Scenario	1		2		3		4	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes
ω	.	0.22	.	0.22	.	0.23	.	0.23
R_1^2	0.65	0.70	0.35	0.40	0.65	0.70	0.35	0.40
R_{2T}^2	.	0.02	.	0.02	.	0.03	.	0.04
n (Average Cluster Size)	20	20	20	20	40	40	40	40
J (# of Clusters)	40	40	40	40	20	20	20	20
SE based on empirical distribution	0.077	0.075	0.085	0.083	0.103	0.101	0.107	0.105
SE calculated from formula	0.037	0.074	0.051	0.082	0.037	0.101	0.051	0.106
Absolute difference in standard errors	-0.040	-0.001	-0.034	-0.001	-0.065	0.000	-0.056	0.001
Relative difference in standard errors (%)	-0.52	-0.02	-0.40	-0.02	-0.64	0.00	-0.52	0.01
Coverage rate of 95% CI	0.667	0.941	0.771	0.945	0.537	0.944	0.664	0.950
Type I error rate estimated from simulation	0.323	0.061	0.218	0.057	0.457	0.069	0.330	0.064
Type I error rate calculated from formula	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
Absolute difference in Type I error rate	-0.273	-0.011	-0.168	-0.007	-0.407	-0.019	-0.280	-0.014
Relative difference in Type I error rate (%)	-84.50	-18.55	-77.07	-11.66	-89.05	-27.85	-84.82	-22.11

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The effect size difference = 0. The intraclass correlation, ρ = 0.2. The proportion of clusters assigned to the treatment group, P = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S12

Coverage of 95% Confidence Interval and Power from Monte Carlo Simulation and the Formulas for a Continuous Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.2 in the Data Generation Model

Scenario	1		2		3		4		5		6		7		8	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Effect Size Difference	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10
ω	.	0.22	.	0.22	.	0.22	.	0.22	.	0.23	.	0.23	.	0.23	.	0.23
R_1^2	0.66	0.70	0.36	0.40	0.65	0.70	0.35	0.40	0.66	0.70	0.36	0.40	0.66	0.70	0.36	0.40
R_{2T}^2	.	0.24	.	0.25	.	0.07	.	0.07	.	0.24	.	0.25	.	0.08	.	0.08
n (Average Cluster Size)	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40
J (# of Clusters)	40	40	40	40	40	40	40	40	20	20	20	20	20	20	20	20
SE based on empirical distribution	0.068	0.067	0.077	0.076	0.074	0.072	0.082	0.081	0.090	0.088	0.096	0.095	0.098	0.096	0.105	0.103
SE calculated from formula	0.037	0.067	0.050	0.075	0.037	0.073	0.051	0.080	0.037	0.091	0.050	0.096	0.037	0.098	0.051	0.104
Absolute difference in standard errors	-0.032	0.000	-0.026	-0.001	-0.037	0.000	-0.031	-0.001	-0.053	0.002	-0.046	0.001	-0.061	0.002	-0.054	0.001
Relative difference in standard errors (%)	-0.46	0.00	-0.34	-0.02	-0.50	0.00	-0.38	-0.01	-0.59	0.03	-0.48	0.01	-0.62	0.02	-0.52	0.01
Coverage rate of 95% CI	0.710	0.946	0.811	0.944	0.675	0.947	0.777	0.944	0.592	0.946	0.711	0.946	0.556	0.948	0.674	0.947
Power estimated from simulation	0.964	0.842	0.898	0.745	0.643	0.295	0.490	0.238	0.922	0.627	0.843	0.568	0.634	0.198	0.517	0.180
Power calculated from formula	1.000	0.829	0.977	0.741	0.774	0.276	0.500	0.230	1.000	0.553	0.977	0.507	0.767	0.162	0.503	0.151
Absolute difference in power	0.036	-0.013	0.079	-0.005	0.130	-0.019	0.010	-0.008	0.078	-0.073	0.134	-0.061	0.133	-0.036	-0.014	-0.029
Relative difference in power (%)	3.73	-1.59	8.83	-0.63	20.25	-6.48	2.07	-3.44	8.44	-11.70	15.91	-10.71	20.98	-17.96	-2.66	-16.32

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The intraclass correlation, $\rho = 0.2$. The proportion of clusters assigned to the treatment group, $P = 0.5$. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at $\alpha = 0.05$. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S13

Coverage of 95% Confidence Interval and Type I Error Rate from Monte Carlo Simulation and the Formulas for a Binary Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.4 in the Data Generation Model

Scenario	1		2		3		4	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes
ω	.	0.43	.	0.42	.	0.45	.	0.44
R_1^2	0.68	0.70	0.38	0.40	0.68	0.70	0.38	0.40
R_{2T}^2	.	0.02	.	0.04	.	0.04	.	0.05
n (Average Cluster Size)	20	20	20	20	40	40	40	40
J (# of Clusters)	40	40	40	40	20	20	20	20
SE based on empirical distribution	0.115	0.115	0.136	0.136	0.149	0.149	0.167	0.167
SE calculated from formula	0.072	0.115	0.100	0.133	0.072	0.148	0.100	0.162
Absolute difference in standard errors	-0.043	0.000	-0.036	-0.003	-0.077	-0.001	-0.067	-0.004
Relative difference in standard errors (%)	-0.37	0.00	-0.27	-0.02	-0.52	-0.01	-0.40	-0.03
Coverage rate of 95% CI	0.787	0.949	0.854	0.947	0.668	0.949	0.773	0.948
Type I error rate estimated from simulation	0.203	0.056	0.136	0.056	0.327	0.065	0.222	0.064
Type I error rate calculated from formula	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
Absolute difference in Type I error rate	-0.153	-0.006	-0.086	-0.006	-0.277	-0.015	-0.172	-0.014
Relative difference in Type I error rate (%)	-75.40	-11.34	-63.10	-10.39	-84.69	-22.48	-77.47	-22.35

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The effect size difference = 0. The intraclass correlation, ρ = 0.2. The proportion of clusters assigned to the treatment group, P = 0.5. The proportion of the individuals in one moderator subgroup, Q = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S14

Coverage of 95% Confidence Interval and Power from Monte Carlo Simulation and the Formulas for a Binary Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.4 in the Data Generation Model

Scenario	1		2		3		4		5		6		7		8	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Effect Size Difference	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10
ω	.	0.43	.	0.43	.	0.43	.	0.42	.	0.45	.	0.44	.	0.44	.	0.44
R_1^2	0.68	0.70	0.38	0.40	0.68	0.70	0.38	0.40	0.68	0.70	0.38	0.40	0.68	0.70	0.38	0.40
R_{2T}^2	.	0.13	.	0.15	.	0.05	.	0.06	.	0.14	.	0.15	.	0.06	.	0.07
n (Average Cluster Size)	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40
J (# of Clusters)	40	40	40	40	40	40	40	40	20	20	20	20	20	20	20	20
SE based on empirical distribution	0.111	0.111	0.132	0.132	0.114	0.114	0.134	0.134	0.141	0.141	0.159	0.159	0.148	0.148	0.164	0.164
SE calculated from formula	0.072	0.111	0.100	0.130	0.072	0.114	0.100	0.132	0.072	0.142	0.100	0.157	0.072	0.146	0.100	0.161
Absolute difference in standard errors	-0.039	0.000	-0.032	-0.002	-0.042	0.001	-0.035	-0.002	-0.069	0.001	-0.059	-0.002	-0.076	-0.001	-0.064	-0.003
Relative difference in standard errors (%)	-0.35	0.00	-0.24	-0.01	-0.37	0.01	-0.26	-0.02	-0.49	0.01	-0.37	-0.01	-0.51	-0.01	-0.39	-0.02
Coverage rate of 95% CI	0.795	0.944	0.871	0.949	0.791	0.949	0.860	0.948	0.693	0.948	0.796	0.948	0.675	0.948	0.784	0.948
Power estimated from simulation	0.689	0.447	0.491	0.334	0.362	0.146	0.240	0.122	0.665	0.322	0.493	0.263	0.424	0.126	0.297	0.115
Power calculated from formula	0.795	0.424	0.517	0.324	0.294	0.141	0.174	0.117	0.809	0.277	0.514	0.228	0.294	0.102	0.174	0.093
Absolute difference in power	0.107	-0.023	0.026	-0.009	-0.068	-0.005	-0.066	-0.006	0.145	-0.045	0.022	-0.036	-0.131	-0.024	-0.123	-0.023
Relative difference in power (%)	15.47	-5.25	5.23	-2.84	-18.82	-3.15	-27.47	-4.68	21.79	-14.10	4.44	-13.55	-30.77	-18.89	-41.34	-19.55

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The intraclass correlation, ρ = 0.2. The proportion of clusters assigned to the treatment group, P = 0.5. The proportion of the individuals in one moderator subgroup, Q = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S15

Coverage of 95% Confidence Interval and Type I Error Rate from Monte Carlo Simulation and the Formulas for a Continuous Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.4 in the Data Generation Model

Scenario	1		2		3		4	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes
ω	.	0.43	.	0.43	.	0.45	.	0.45
R_1^2	0.60	0.70	0.30	0.40	0.61	0.70	0.31	0.39
R_{2T}^2	.	0.01	.	0.02	.	0.03	.	0.03
n (Average Cluster Size)	20	20	20	20	40	40	40	40
J (# of Clusters)	40	40	40	40	20	20	20	20
SE based on empirical distribution	0.102	0.098	0.107	0.105	0.139	0.136	0.142	0.140
SE calculated from formula	0.040	0.099	0.053	0.104	0.040	0.136	0.053	0.141
Absolute difference in standard errors	-0.062	0.001	-0.055	0.000	-0.099	0.000	-0.090	0.001
Relative difference in standard errors (%)	-0.61	0.01	-0.51	0.00	-0.71	0.00	-0.63	0.01
Coverage rate of 95% CI	0.560	0.948	0.676	0.947	0.434	0.949	0.543	0.946
Type I error rate estimated from simulation	0.426	0.057	0.311	0.057	0.561	0.065	0.451	0.064
Type I error rate calculated from formula	0.050	0.050	0.050	0.050	0.051	0.050	0.050	0.050
Absolute difference in Type I error rate	-0.376	-0.007	-0.261	-0.007	-0.510	-0.015	-0.401	-0.014
Relative difference in Type I error rate (%)	-88.27	-11.50	-83.89	-11.96	-90.97	-23.38	-88.90	-22.11

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The effect size difference = 0. The intraclass correlation, ρ = 0.2. The proportion of clusters assigned to the treatment group, P = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S16

Coverage of 95% Confidence Interval and Power from Monte Carlo Simulation and the Formulas for a Continuous Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.4 in the Data Generation Model

Scenario	1		2		3		4		5		6		7		8	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Effect Size Difference	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10
ω	.	0.43	.	0.43	.	0.43	.	0.43	.	0.44	.	0.45	.	0.45	.	0.44
R_1^2	0.61	0.70	0.31	0.40	0.60	0.70	0.30	0.40	0.62	0.70	0.32	0.39	0.61	0.70	0.31	0.40
R_{2T}^2	.	0.12	.	0.12	.	0.04	.	0.04	.	0.13	.	0.13	.	0.05	.	0.05
n (Average Cluster Size)	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40
J (# of Clusters)	40	40	40	40	40	40	40	40	20	20	20	20	20	20	20	20
SE based on empirical distribution	0.097	0.093	0.101	0.099	0.101	0.097	0.105	0.103	0.130	0.127	0.134	0.132	0.135	0.133	0.141	0.138
SE calculated from formula	0.039	0.094	0.052	0.100	0.040	0.098	0.053	0.103	0.039	0.130	0.052	0.134	0.040	0.135	0.053	0.139
Absolute difference in standard errors	-0.057	0.000	-0.049	0.001	-0.061	0.001	-0.052	0.001	-0.090	0.002	-0.081	0.002	-0.096	0.002	-0.088	0.000
Relative difference in standard errors (%)	-0.59	0.00	-0.48	0.01	-0.60	0.01	-0.50	0.01	-0.70	0.02	-0.61	0.02	-0.71	0.02	-0.63	0.00
Coverage rate of 95% CI	0.586	0.942	0.694	0.947	0.561	0.946	0.677	0.948	0.460	0.944	0.564	0.945	0.439	0.949	0.549	0.947
Power estimated from simulation	0.893	0.583	0.822	0.520	0.611	0.192	0.489	0.165	0.834	0.362	0.770	0.349	0.654	0.136	0.548	0.133
Power calculated from formula	0.999	0.551	0.967	0.499	0.708	0.171	0.451	0.152	0.999	0.306	0.970	0.298	0.738	0.112	0.480	0.106
Absolute difference in power	0.107	-0.033	0.146	-0.021	0.096	-0.021	-0.038	-0.013	0.165	-0.055	0.200	-0.051	0.084	-0.024	-0.068	-0.027
Relative difference in power (%)	11.93	-5.61	17.75	-3.99	15.75	-11.05	-7.80	-7.89	19.78	-15.26	25.98	-14.59	12.83	-17.48	-12.34	-20.25

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The intraclass correlation, $\rho = 0.2$. The proportion of clusters assigned to the treatment group, $P = 0.5$. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at $\alpha = 0.05$. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S17

Coverage of 95% Confidence Interval and Type I Error Rate from Monte Carlo Simulation and the Formulas for a Binary Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.6 in the Data Generation Model

Scenario	1		2		3		4	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes
ω	.	0.62	.	0.61	.	0.62	.	0.62
R_1^2	0.66	0.70	0.36	0.40	0.66	0.70	0.36	0.40
R_{2T}^2	.	0.02	.	0.03	.	0.03	.	0.04
n (Average Cluster Size)	20	20	20	20	40	40	40	40
J (# of Clusters)	40	40	40	40	20	20	20	20
SE based on empirical distribution	0.133	0.133	0.151	0.151	0.174	0.174	0.190	0.190
SE calculated from formula	0.073	0.130	0.101	0.147	0.073	0.170	0.101	0.182
Absolute difference in standard errors	-0.060	-0.002	-0.050	-0.005	-0.101	-0.004	-0.090	-0.008
Relative difference in standard errors (%)	-0.45	-0.02	-0.33	-0.03	-0.58	-0.02	-0.47	-0.04
Coverage rate of 95% CI	0.729	0.943	0.815	0.943	0.604	0.945	0.714	0.944
Type I error rate estimated from simulation	0.258	0.060	0.175	0.060	0.390	0.066	0.281	0.065
Type I error rate calculated from formula	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
Absolute difference in Type I error rate	-0.208	-0.010	-0.125	-0.010	-0.340	-0.016	-0.231	-0.015
Relative difference in Type I error rate (%)	-80.65	-16.39	-71.46	-16.94	-87.15	-24.11	-82.19	-23.55

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The effect size difference = 0. The intraclass correlation, ρ = 0.2. The proportion of clusters assigned to the treatment group, P = 0.5. The proportion of the individuals in one moderator subgroup, Q = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S18

Coverage of 95% Confidence Interval and Power from Monte Carlo Simulation and the Formulas for a Binary Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.6 in the Data Generation Model

Scenario	1		2		3		4		5		6		7		8	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Effect Size Difference	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10
ω	.	0.62	.	0.61	.	0.62	.	0.61	.	0.62	.	0.62	.	0.62	.	0.61
R_1^2	0.67	0.70	0.37	0.40	0.66	0.70	0.36	0.40	0.67	0.70	0.37	0.40	0.67	0.70	0.37	0.40
R_{2T}^2	.	0.09	.	0.10	.	0.03	.	0.04	.	0.09	.	0.10	.	0.05	.	0.05
n (Average Cluster Size)	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40
J (# of Clusters)	40	40	40	40	40	40	40	40	20	20	20	20	20	20	20	20
SE based on empirical distribution	0.129	0.129	0.148	0.148	0.132	0.132	0.146	0.146	0.169	0.169	0.185	0.185	0.174	0.174	0.186	0.186
SE calculated from formula	0.073	0.127	0.101	0.144	0.073	0.130	0.101	0.146	0.073	0.165	0.101	0.178	0.073	0.169	0.101	0.181
Absolute difference in standard errors	-0.056	-0.002	-0.047	-0.004	-0.058	-0.002	-0.046	-0.001	-0.096	-0.004	-0.084	-0.007	-0.101	-0.005	-0.086	-0.005
Relative difference in standard errors (%)	-0.43	-0.02	-0.32	-0.03	-0.44	-0.02	-0.31	0.00	-0.57	-0.02	-0.45	-0.04	-0.58	-0.03	-0.46	-0.03
Coverage rate of 95% CI	0.744	0.945	0.823	0.941	0.732	0.946	0.829	0.947	0.616	0.942	0.732	0.945	0.605	0.944	0.722	0.944
Power estimated from simulation	0.653	0.358	0.489	0.281	0.382	0.128	0.254	0.103	0.636	0.234	0.509	0.215	0.466	0.108	0.331	0.099
Power calculated from formula	0.778	0.336	0.501	0.271	0.267	0.115	0.165	0.102	0.775	0.208	0.511	0.189	0.281	0.088	0.165	0.082
Absolute difference in power	0.124	-0.022	0.012	-0.010	-0.115	-0.013	-0.088	-0.001	0.139	-0.026	0.003	-0.025	-0.184	-0.020	-0.166	-0.017
Relative difference in power (%)	19.03	-6.07	2.52	-3.58	-30.06	-9.91	-34.87	-1.34	21.78	-10.95	0.52	-11.86	-39.57	-18.53	-50.06	-17.26

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The intraclass correlation, $\rho = 0.2$. The proportion of clusters assigned to the treatment group, $P = 0.5$. The proportion of the individuals in one moderator subgroup, $Q = 0.5$. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at $\alpha = 0.05$. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S19

Coverage of 95% Confidence Interval and Type I Error Rate from Monte Carlo Simulation and the Formulas for a Continuous Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.6 in the Data Generation Model

Scenario	1		2		3		4	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes
ω	.	0.63	.	0.62	.	0.63	.	0.63
R_1^2	0.55	0.70	0.26	0.40	0.56	0.69	0.26	0.39
R_{2T}^2	.	0.01	.	0.02	.	0.03	.	0.03
n (Average Cluster Size)	20	20	20	20	40	40	40	40
J (# of Clusters)	40	40	40	40	20	20	20	20
SE based on empirical distribution	0.122	0.117	0.128	0.124	0.167	0.164	0.171	0.168
SE calculated from formula	0.042	0.117	0.055	0.121	0.042	0.160	0.054	0.164
Absolute difference in standard errors	-0.079	-0.001	-0.073	-0.003	-0.125	-0.004	-0.116	-0.004
Relative difference in standard errors (%)	-0.65	-0.01	-0.57	-0.02	-0.75	-0.02	-0.68	-0.02
Coverage rate of 95% CI	0.502	0.948	0.601	0.942	0.381	0.943	0.480	0.946
Type I error rate estimated from simulation	0.486	0.055	0.388	0.059	0.615	0.066	0.514	0.065
Type I error rate calculated from formula	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
Absolute difference in Type I error rate	-0.436	-0.005	-0.338	-0.009	-0.565	-0.016	-0.464	-0.015
Relative difference in Type I error rate (%)	-89.69	-9.07	-87.11	-15.82	-91.82	-23.87	-90.25	-22.58

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The effect size difference = 0. The intraclass correlation, ρ = 0.2. The proportion of clusters assigned to the treatment group, P = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S20

Coverage of 95% Confidence Interval and Power from Monte Carlo Simulation and the Formulas for a Continuous Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.6 in the Data Generation Model

Scenario	1		2		3		4		5		6		7		8	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Effect Size Difference	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10
ω	.	0.63	.	0.63	.	0.63	.	0.63	.	0.63	.	0.63	.	0.63	.	0.63
R_1^2	0.57	0.70	0.27	0.40	0.56	0.70	0.26	0.40	0.57	0.69	0.27	0.39	0.56	0.69	0.26	0.39
R_{2T}^2	.	0.08	.	0.08	.	0.03	.	0.03	.	0.09	.	0.09	.	0.04	.	0.05
n (Average Cluster Size)	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40
J (# of Clusters)	40	40	40	40	40	40	40	40	20	20	20	20	20	20	20	20
SE based on empirical distribution	0.117	0.113	0.122	0.119	0.121	0.116	0.126	0.123	0.161	0.158	0.166	0.162	0.168	0.165	0.170	0.167
SE calculated from formula	0.042	0.113	0.054	0.118	0.042	0.116	0.055	0.121	0.041	0.155	0.054	0.159	0.042	0.159	0.054	0.162
Absolute difference in standard errors	-0.076	0.000	-0.068	-0.001	-0.079	0.000	-0.072	-0.002	-0.120	-0.003	-0.111	-0.003	-0.127	-0.006	-0.115	-0.005
Relative difference in standard errors (%)	-0.64	0.00	-0.56	0.00	-0.65	0.00	-0.57	-0.02	-0.74	-0.02	-0.67	-0.02	-0.75	-0.04	-0.68	-0.03
Coverage rate of 95% CI	0.515	0.944	0.617	0.947	0.508	0.944	0.606	0.942	0.391	0.942	0.492	0.937	0.382	0.939	0.474	0.944
Power estimated from simulation	0.840	0.424	0.773	0.398	0.609	0.149	0.516	0.138	0.803	0.268	0.743	0.258	0.666	0.116	0.582	0.112
Power calculated from formula	0.997	0.404	0.959	0.382	0.661	0.135	0.454	0.130	0.998	0.229	0.958	0.224	0.652	0.091	0.460	0.091
Absolute difference in power	0.158	-0.020	0.186	-0.016	0.052	-0.014	-0.062	-0.009	0.195	-0.039	0.215	-0.034	-0.014	-0.025	-0.122	-0.021
Relative difference in power (%)	18.76	-4.65	24.01	-3.94	8.55	-9.16	-12.04	-6.35	24.25	-14.69	28.97	-13.03	-2.04	-21.49	-20.90	-18.48

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The intraclass correlation, $\rho = 0.2$. The proportion of clusters assigned to the treatment group, $P = 0.5$. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at $\alpha = 0.05$. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S21

Coverage of 95% Confidence Interval and Type I Error Rate from Monte Carlo Simulation and the Formulas for a Binary Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.8 in the Data Generation Model

Scenario	1		2		3		4	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes
ω	.	0.77	.	0.76	.	0.75	.	0.74
R_1^2	0.65	0.70	0.35	0.40	0.65	0.70	0.35	0.40
R_{2T}^2	.	0.02	.	0.02	.	0.03	.	0.04
n (Average Cluster Size)	20	20	20	20	40	40	40	40
J (# of Clusters)	40	40	40	40	20	20	20	20
SE based on empirical distribution	0.147	0.147	0.164	0.164	0.197	0.197	0.208	0.208
SE calculated from formula	0.075	0.141	0.102	0.156	0.075	0.184	0.102	0.195
Absolute difference in standard errors	-0.073	-0.006	-0.062	-0.008	-0.123	-0.013	-0.106	-0.013
Relative difference in standard errors (%)	-0.49	-0.04	-0.38	-0.05	-0.62	-0.07	-0.51	-0.06
Coverage rate of 95% CI	0.690	0.939	0.785	0.940	0.554	0.934	0.673	0.938
Type I error rate estimated from simulation	0.301	0.057	0.204	0.058	0.441	0.067	0.320	0.064
Type I error rate calculated from formula	0.050	0.050	0.050	0.050	0.051	0.050	0.050	0.050
Absolute difference in Type I error rate	-0.250	-0.007	-0.154	-0.008	-0.391	-0.016	-0.270	-0.014
Relative difference in Type I error rate (%)	-83.36	-12.58	-75.46	-13.49	-88.55	-24.69	-84.35	-22.12

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The effect size difference = 0. The intraclass correlation, ρ = 0.2. The proportion of clusters assigned to the treatment group, P = 0.5. The proportion of the individuals in one moderator subgroup, Q = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S22

Coverage of 95% Confidence Interval and Power from Monte Carlo Simulation and the Formulas for a Binary Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.8 in the Data Generation Model

Scenario	1		2		3		4		5		6		7		8	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Effect Size Difference	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10
ω	.	0.77	.	0.75	.	0.77	.	0.75	.	0.75	.	0.74	.	0.76	.	0.75
R_1^2	0.65	0.70	0.35	0.40	0.65	0.70	0.35	0.40	0.66	0.70	0.36	0.40	0.65	0.70	0.35	0.40
R_{2T}^2	.	0.07	.	0.07	.	0.03	.	0.03	.	0.08	.	0.08	.	0.04	.	0.05
n (Average Cluster Size)	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40
J (# of Clusters)	40	40	40	40	40	40	40	40	20	20	20	20	20	20	20	20
SE based on empirical distribution	0.144	0.144	0.161	0.161	0.146	0.146	0.163	0.163	0.194	0.194	0.206	0.206	0.196	0.196	0.211	0.210
SE calculated from formula	0.074	0.138	0.102	0.154	0.075	0.141	0.102	0.156	0.074	0.181	0.102	0.192	0.075	0.184	0.102	0.194
Absolute difference in standard errors	-0.070	-0.006	-0.060	-0.008	-0.071	-0.005	-0.062	-0.008	-0.120	-0.013	-0.104	-0.014	-0.121	-0.012	-0.109	-0.016
Relative difference in standard errors (%)	-0.48	-0.04	-0.37	-0.05	-0.49	-0.04	-0.38	-0.05	-0.62	-0.07	-0.51	-0.07	-0.62	-0.06	-0.52	-0.08
Coverage rate of 95% CI	0.695	0.938	0.795	0.936	0.690	0.945	0.784	0.941	0.556	0.932	0.676	0.937	0.555	0.937	0.673	0.935
Power estimated from simulation	0.641	0.298	0.495	0.249	0.403	0.116	0.280	0.100	0.635	0.201	0.507	0.185	0.500	0.095	0.376	0.094
Power calculated from formula	0.765	0.293	0.508	0.250	0.276	0.109	0.163	0.095	0.763	0.183	0.487	0.164	0.279	0.083	0.162	0.077
Absolute difference in power	0.124	-0.005	0.013	0.001	-0.127	-0.007	-0.117	-0.005	0.128	-0.018	-0.019	-0.020	-0.221	-0.012	-0.214	-0.017
Relative difference in power (%)	19.27	-1.82	2.66	0.48	-31.53	-5.77	-41.81	-4.75	20.17	-9.17	-3.79	-11.04	-44.25	-12.52	-56.87	-18.19

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The intraclass correlation, ρ = 0.2. The proportion of clusters assigned to the treatment group, P = 0.5. The proportion of the individuals in one moderator subgroup, Q = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S23

Coverage of 95% Confidence Interval and Type I Error Rate from Monte Carlo Simulation and the Formulas for a Continuous Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.8 in the Data Generation Model

Scenario	1		2		3		4	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes
ω	.	0.78	.	0.77	.	0.76	.	0.76
R_1^2	0.51	0.70	0.21	0.39	0.51	0.69	0.21	0.39
R_{2T}^2	.	0.01	.	0.01	.	0.03	.	0.03
n (Average Cluster Size)	20	20	20	20	40	40	40	40
J (# of Clusters)	40	40	40	40	20	20	20	20
SE based on empirical distribution	0.138	0.132	0.143	0.139	0.194	0.190	0.194	0.190
SE calculated from formula	0.044	0.129	0.056	0.133	0.044	0.176	0.056	0.179
Absolute difference in standard errors	-0.094	-0.004	-0.087	-0.006	-0.150	-0.015	-0.138	-0.011
Relative difference in standard errors (%)	-0.68	-0.03	-0.61	-0.04	-0.77	-0.08	-0.71	-0.06
Coverage rate of 95% CI	0.476	0.943	0.568	0.937	0.351	0.929	0.437	0.936
Type I error rate estimated from simulation	0.514	0.054	0.423	0.059	0.645	0.068	0.559	0.064
Type I error rate calculated from formula	0.051	0.050	0.050	0.050	0.050	0.050	0.050	0.050
Absolute difference in Type I error rate	-0.464	-0.004	-0.373	-0.009	-0.595	-0.018	-0.509	-0.014
Relative difference in Type I error rate (%)	-90.14	-6.95	-88.16	-15.20	-92.24	-26.36	-91.05	-22.12

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The effect size difference = 0. The intraclass correlation, ρ = 0.2. The proportion of clusters assigned to the treatment group, P = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.

Table S24

Coverage of 95% Confidence Interval and Power from Monte Carlo Simulation and the Formulas for a Continuous Moderator when the Heterogeneity Coefficient (ω) of the Moderator Is Set as 0.8 in the Data Generation Model

Scenario	1		2		3		4		5		6		7		8	
Randomly Varying Slope Model	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Effect Size Difference	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.10	0.10	0.10	0.10
ω	.	0.78	.	0.78	.	0.78	.	0.77	.	0.76	.	0.76	.	0.76	.	0.76
R_1^2	0.52	0.70	0.22	0.39	0.51	0.70	0.21	0.40	0.52	0.69	0.22	0.39	0.52	0.69	0.22	0.39
R_{2T}^2	.	0.07	.	0.07	.	0.03	.	0.03	.	0.07	.	0.07	.	0.04	.	0.04
n (Average Cluster Size)	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40
J (# of Clusters)	40	40	40	40	40	40	40	40	20	20	20	20	20	20	20	20
SE based on empirical distribution	0.137	0.132	0.138	0.134	0.139	0.134	0.144	0.139	0.187	0.183	0.189	0.185	0.191	0.187	0.193	0.190
SE calculated from formula	0.044	0.126	0.056	0.130	0.044	0.128	0.056	0.132	0.044	0.172	0.056	0.175	0.044	0.175	0.056	0.177
Absolute difference in standard errors	-0.093	-0.006	-0.083	-0.004	-0.094	-0.005	-0.088	-0.006	-0.143	-0.011	-0.133	-0.011	-0.147	-0.013	-0.138	-0.012
Relative difference in standard errors (%)	-0.68	-0.04	-0.60	-0.03	-0.68	-0.04	-0.61	-0.05	-0.77	-0.06	-0.71	-0.06	-0.77	-0.07	-0.71	-0.06
Coverage rate of 95% CI	0.472	0.933	0.580	0.939	0.469	0.937	0.567	0.935	0.364	0.933	0.445	0.936	0.351	0.935	0.444	0.934
Power estimated from simulation	0.811	0.345	0.746	0.318	0.609	0.125	0.527	0.123	0.783	0.214	0.718	0.207	0.677	0.101	0.609	0.101
Power calculated from formula	0.996	0.342	0.943	0.320	0.612	0.119	0.422	0.114	0.995	0.195	0.940	0.187	0.613	0.084	0.432	0.084
Absolute difference in power	0.184	-0.004	0.198	0.002	0.003	-0.006	-0.104	-0.009	0.212	-0.019	0.222	-0.020	-0.064	-0.017	-0.178	-0.017
Relative difference in power (%)	22.73	-1.02	26.49	0.72	0.46	-5.19	-19.80	-7.34	27.12	-8.72	30.90	-9.75	-9.43	-17.04	-29.15	-16.98

Note. Results were based on 10,000 replications. R_1^2 is the proportion of variance at level 1 explained by level-1 covariates. R_{2T}^2 is the proportion of variance in the treatment effect explained by covariates. The intraclass correlation, ρ = 0.2. The proportion of clusters assigned to the treatment group, *P* = 0.5. The 95% confidence intervals (CI) were constructed from the standard error (SE) that was calculated from formulas at α = 0.05. Coverage rates were calculated based on percent of times that the 95% confidence intervals included the true moderator effect.